

Mathematical Network

International groups and congresses



Everything is mathematical

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Guillermo Curbera

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Preface

Can we get by without mathematicians yet still keep mathematics alive? This question may arise from the fact that a modern mathematician uses computers to carry out his or her tasks. Specifically, there are mathematical results that have been proven, not by a person, but by a computer. (Among them is the four-colour theorem, which states that four colours are enough to colour any map such that adjacent regions have different colours.) In addition, an active area of mathematical research is the automatic demonstration of theorems, which seeks to reduce the complicated process of mental reasoning that establishes the truth of a mathematical fact to a 'simple' verification by computer. The fact that all mathematical results and their demonstrations are very long and somewhat complicated strings of signs and symbols points in the same direction. If we set supercomputers the task of producing all the possible sequences of signs and symbols, eventually all the results in mathematics and their proofs would appear eventually.

But what about people? Where does this leave the mathematicians? French mathematician Jacques Hadamard wrote an essay (at the time when, as a Jew, he had taken refuge from the Nazis in the US during the World War II) in which he investigated the elements of invention in mathematics, dissecting the role of creativity from the delicate mental mechanisms. The book is still read and studied today. Will the course of history relegate the book to a mere memory, along with mathematicians?

For now at least, the answer seems to be no. Firstly, because the practice of mathematics continues to require creativity and the originality of human thought – the proof of the four-colour theorem is the concept of a human mind; the computer only carried out the tedious verifications. But there is another reason: mathematics has a very intense collective component. The evaluation of a mathematical result, the determination of the most significant lines of research, are not tasks that are carried out by individuals. In this sense, the internal organisation and life of mathematical researchers as a group have an enormous influence on the development of the subject.

This book presents a vision of the internal life of the international mathematical community. We will follow the history of the International Congresses of Mathematicians, which have been, and continue to be, the Olympics of mathematics. (The meetings began in 1897, the year after its sporting cousin.)

Explaining the reason for the unique values of these congresses requires a certain amount of investigation into the various characters that reside within mathematics;

we do this in the book's first chapter. In the following chapters we will visit the congresses before World War I and the turbulent period between the wars. We will have an intermission in Chapter 4, in which we will discuss the most prestigious prize in mathematics – the Fields Medal. Returning to the course of history, we will look at the post-World War II congresses, those held during the Cold War and those in our current globalised world. We conclude with a final, more technical chapter, where we discuss what is considered the most important mathematical problem pending resolution, the Riemann hypothesis, which has been awaiting a solution for more than 150 years and promises the mathematician who resolves it a cool \$1 million and, even more valuable, a place in history.

Chapter 1

The Soul of Mathematics

It is true that the majority of ideas in our science have come from, and matured in, the silence of the study; no other science, except perhaps philosophy, is as hermitic and reclusive as mathematics. Even so, in the heart of the mathematician resides the need to communicate and express himself to his colleagues.

Each one of us knows very well, through personal experience, how stimulating scientific exchange can be.

These words may surprise anyone who has not experienced, personally or by other means, creation in mathematics. They will surely agree with the first component: mathematics is, or appears to be, a reclusive activity, sometimes to the point of being hermitic. This marries up with the common image of the mathematician as a reserved, even withdrawn, and absent-minded person. This is demonstrated by Titus Livius' tale of the capture of Syracuse by the consul Marcellus, which tells of the death of the ancient world's greatest mathematician, Archimedes, at the hands of the Roman legionaries, "while he leant over a drawing he had made in the sand". We imagine Archimedes lost in his thoughts, ignorant of the cruelty that threatened him.

On the other hand, the second factor can be surprising – the need to communicate a passion for the exchange of ideas through words. Like a tennis player serving a ball and waiting for it to come back, the mathematician, upon creating an idea or glimpsing the solution to a problem, needs to demonstrate it, to present it, in order for it to be returned with comments, criticisms and praise. It is public judgement that certifies the truth; it is multiple opinions and other people's perspective that inform us of the success or failure of our work.

Someone who explained this duality of reclusion and communication that resides within a mathematician must have had a good understanding of this. That someone

was Adolf Hurwitz, a German mathematician based in Zurich, who dedicated his life, astride the 19th and 20th centuries, to mathematical investigation and also, as we will see later, to driving the growth of communication between mathematicians.

Another duality pulsates, and practically resounds, within mathematics. In 1960 Eugene Wigner, winner of the Nobel Prize for physics, published a well-known article, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", in which he discussed that very phenomenon. Eugene Wigner opened with this observation:

"The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and there is no rational explanation for it."

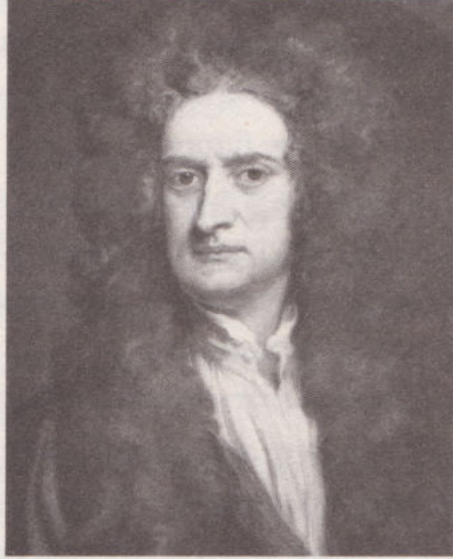
We should clarify that the unreasonable thing is not that mathematics explains the natural world. There is mathematics that is made specifically to deal with real life. What is harder to accept is that the mathematics that has been kept absolutely pure, free of any attempt at utility, and which has been created only subject to its own internal rules of formation and coherence, is subsequently capable, after its birth, of explaining everyday phenomena. You may argue, quite rightly, that a mathematician feeds his or her imagination and primes his or her creative dreams with the reality that surrounds him or her, in which he or she is immersed. But the creativity only goes that far, as once the fuse to the solution to the problem has been lit, the pieces fit together like large protein molecules, without any room for error or whim. And, surprisingly, the result ends up unravelling the most prosaic of events, like a ripple in a pond or an eddy in a stream.

But the mathematician is not unique, just as the Greek heroes were not: they demonstrated a huge diversity of talents and intentions. Thus, among the mathematicians there is a vast range of attitudes to that reality which, like a distant murmur, never cease to remind us that, eventually, our pure theories will end up helping to depict our world. You do not have to delve too deeply into the list of successes to find these antithetical attitudes, even residing in the same people. History's three greatest mathematicians, Archimedes, Newton and Gauss, were all preachers of the most immaculate purity (remember Archimedes' measurement of the sphere, Newton's creation of infinitesimal calculus and Gauss' invention of modular arithmetic) and, at the same time, avid engineers or applied scientists (Archimedes' study of the lever, the unveiling of the tides by Newton, the creation of the telegraph by Gauss).

Another tension in the soul of mathematics, this time between the general and the specific, is balanced out by the opposition between pure mathematics and applied mathematics.



Archimedes of Syracuse.



Isaac Newton.



Carl Friedrich Gauss.

Future and past is another very powerful internal duality of mathematics. Like all sciences, mathematics works by looking to the future, glimpsing new realities and new problems that arise as it advances. But, unlike most sciences, mathematics is very

aware of its past. Not only through honouring the work and the people who have gone before us, but, mainly, because active mathematics continues to draw inspiration from the great works of the past. In other scientific disciplines, an article from over ten years ago, whether it be 30, 40 or 50 years old, is useless, perhaps even wrong. On the other hand, university libraries keep old mathematics magazines dust-free, as mathematicians continue to consult their articles. This is where mathematics' special dedication and reverence for its past comes from, because it continues to be a force that drives us into the future.

And we can find even more dualities within mathematics, between specialisation in specifics and the global vision, with its thirst for universality; between the Dionysian spirit of creation and heuristics and the Apollonian spirit that relentlessly compels mathematical rigour.

These opposing characteristics, which are also inseparably coupled (reclusion and communication, pure and applied, history and future), reside in the multiform soul of mathematics and are present throughout its history, flourishing wherever we pause to look. All of them interleave to generate in mathematics and its practitioners the strong feeling of community, united by a powerful past and fertilised by an imperious need to share the work. As accustomed as we are to associating the existence of a community identity with religious, political or national groups, it may surprise us to find it with equal intensity in a scientific community.

In this book we propose a journey to contemplate the community spirit and the contradictory souls of mathematics in action. To do so we will follow the course of the International Congresses of Mathematicians (better known by their acronym, ICM).

Since 1897, these congresses have been held, like the Olympics, almost every four years and, like them, they bring together the best in the profession. They feature ground-breaking performances, and the most sought-after prizes are awarded at them. There are many scientific, and non-scientific, congresses, even in mathematics, but these are different.

During the opening ceremony of the international congress that was held in Amsterdam in 1954, the American mathematician Oswald Veblen explained:

“The series of International Congresses are very loosely held together. They are not congresses of mathematics, that highly organised body of knowledge, but of mathematicians, those rather chaotic individuals who create and conserve it.”

Oswald Veblen knew the subject well as he had presided over the previous congress, held at Harvard University in 1950. His interpretation was not an exercise in rhetoric. From their very conception, the international congresses of mathematicians have waved the flag of personal relationships, the exchange between people as a fundamental factor that contributes to the development of mathematics. Moreover, this has been done explicitly. At the first congress, held in Zurich in 1897, it was agreed that the principal objective of the congresses would be to "Promote personal relations among the mathematicians of different countries". It is also true that an explicitly scientific goal was included, but it was secondary.

The mathematical contributions presented at the international congresses form a colourful kaleidoscope of mathematics in the 20th century. They occupy most of the 60 volumes of minutes produced during the 26 congresses held to date. Immersing ourselves in this lush garden of mathematics, which is as fertile as it is technical, could not be further from our purpose here.

The human factor referred to by Oswald Veblen provides the international congresses, aside from their technical aspect, with a human, sociological or cultural aspect, depending on how we look at it, which is the fruit of a spirit of community within mathematics. This is why the congresses reflect the world so well, covering political and economic events, scientific and technological discoveries, social customs, etc. This other face of the congresses is the one we are going to explore, with one goal – to look and to see.

We can look and see how a scientific community works from within, how it makes its decisions and resolves disagreements, how it rewards and how it punishes. The names of the speakers, the titles of the conferences or the lists of the scientific sections can be long, even tedious. But if we look closely, we will see that they show the passing of time in mathematics; we will see how they abandon some subjects and take on new ones; we will see how the titles of the conferences lose their beauty, become jammed with technicalities; we will see some languages, which used to dominate, decline and eventually disappear; we will see how the divisions of mathematics itself (so immutable!) can change, advance, twist and subdivide.

The same is true of the names of the mathematicians and the institutes where they work. We know some of them; others may ring a bell, and many are unknown to us, but those names speak of the places in the world where mathematics is practised, of how mathematicians move around the world, and of what the science is like. But there is more to look at and to see. Around the international congresses

we see the appearance of events, cities, places and languages. All of this shows us the glories and obsessions of those countries, those generations, those times, the changes that they enjoyed or suffered. It allows us to glimpse, through the filter of the mathematical community, the world in which they take place.

We are going to conclude this introduction with an example that underpins the previous explanation. The 1936 international congress was held in Oslo. Among the social events was a fabulous night excursion by boat through the fjords of Oslo, which was attended by some 700 people and which was survived by a famous photo (of the mathematical celebrities of the time having their dinner) and a vivid account (courtesy of Waldo Dunnington, the author of the well-known biography *Carl Friedrich Gauss: Titan of Science*):

“The afternoon and evening of Thursday, 16 July, was dedicated to a boat trip through the Oslo fjord on the *SS. Stavangerfjord*, the Norwegian America Lines’ largest ship. The royal prince and princess participated in the excursion, and at six in the afternoon a banquet was served in four of the boat’s dining rooms. The brief speeches were transmitted to all the dining rooms by means of speakers, and the music of Grieg, Sibelius, Strauss and a march dedicated to Prince Olaf were performed. At night there was a dance and a game of cards, cheerful conversations and memories, with the on-board restaurant and bar in full swing. The boat docked in Oslo at midnight and the taxis waited to take the guests to their hotels.”

Now let’s take a look at some of the more recent congresses, such as Berkeley in 1986, Berlin in 1998, Beijing in 2002 and Madrid in 2006. We no longer find such refined celebrations, but massive ‘parties’ celebrated in the open air and enhanced by music and beer.

How the world has changed! The huge distance that separates these two so-different activities is more than just the passing of time. Mathematicians have gone from being a select few, socially elite and the exclusive owners of sophisticated knowledge, to being a broad, open and mobile group of highly specialised people. The sociology of those who work in science has changed.

Chapter 2

The Most Important Lecture Ever

One mathematics lecture is, beyond doubt, the most important in history. It was given by the German mathematician David Hilbert, from the University of Göttingen, who was, together with the Frenchman Henri Poincaré, of the University of Sorbonne in Paris, the most important mathematician at the time. We will look at its content later, but now let's take a look at the circumstances of the conference. The time and place was Paris at the dawn of the 20th century, on a hot August morning. The occasion was the second International Mathematics Congress.

More than a century later, scientific congresses now seem normal to us, but this was not the case in the early 20th century. Let's cast our minds back and discover when, where and why these scientific congresses came about.



David Hilbert.



Henri Poincaré.

From the isolated sage to the community scientist

Until the 18th century working in science (in mathematics in particular) was generally a passionate but complementary activity, which alone did not provide a living. The situation changed with the rise of the powerful modern states, whose patronage allowed the creation of scientific academies towards the end of the 17th century. Among the first were the Royal Society of London and the Académie Royale des Sciences in Paris, which were created in the 1660s. A few years later, in 1700, the Academy of Berlin was created and in 1724, the Saint Petersburg Academy. These institutions brought together the great mathematicians of the 18th century: Daniel and Nicolaus Bernoulli, Leonhard Euler, Joseph-Louis Lagrange, Jean d'Alembert and many others. These sages generally worked in isolation, communicating by means of letters, which slowly crossed Europe.

And suddenly everything changed. The French Revolution disrupted the already out-of-date order of the Ancien Régime and shook Europe out of its complacency. The profound political, social and economic changes came, as was expected, as they did in education and science. France began to close all its universities, which were deeply rooted in Medieval scholasticism, and created a new type of institution – the École Polytechnique, which brought together education, science and practical applications.

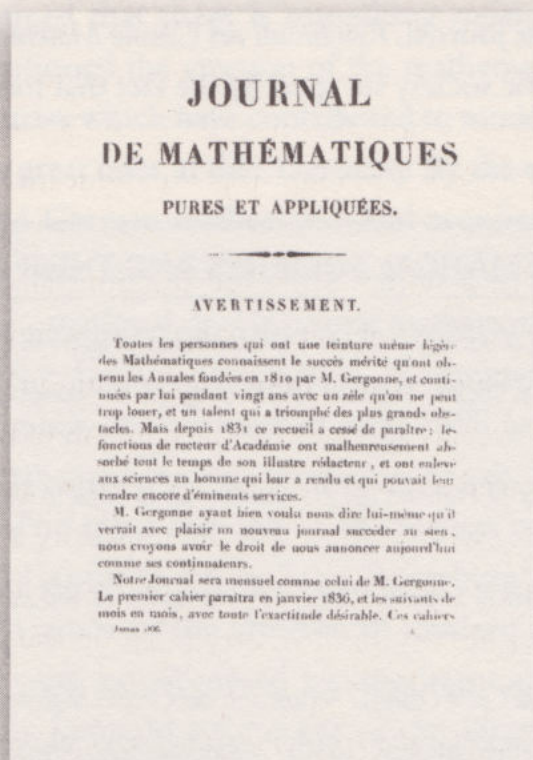
The success of the new system provoked reactions all across Europe, and not only negative ones. The victories of Napoleon's armies were ascribed to the rigorous training of their officials. Thus, Prussian thinker and politician Wilhelm von Humboldt came up with a new university model with the creation of the University of Berlin in 1810, a model that spread to other Prussian universities, and became the ideal that we still use today. Science was transferred to the university, and its development was linked to higher education. Research and teaching were the new, and obligatory, tasks of the university professors. In turn, they found in the universities somewhere that welcomed them.

Thus appeared chairs, new academic positions, specialised libraries and seminars, which allowed the methodical and systematic preparation of students. Just as with the exponential growth of industrial production, science and scientists, especially mathematicians, multiplied.

This was the time when scientific journals specific to mathematics first bloomed. The first scientific journals had appeared during the Scientific Revolution, in the second part of the 17th century, and were published by the academies. Many of

them are renowned. They included the *Journal des Savants*, which appeared in France in 1665, promoted by private editors; the Royal Society's *Philosophical Transactions* founded in London, also in 1665; and the *Acta Eruditorum*, edited in Leipzig since 1682 (where the works of the great mathematician Gottfried Leibniz on differential and integral calculus appeared). During the 18th century, the scientific academies created scientific journals that covered all the major disciplines. But, as the century advanced, the need for specialisation led to the creation of specific journals for each science.

The first in mathematics were the *Annales de mathématiques pures et appliquées*, founded by Joseph Diaz Gergonne in 1810; the *Journal für die reine und angewandte Mathematik*, founded by August Leopold Crelle in 1826; and the *Journal de mathématiques pures et appliquées*, founded by Joseph-Louis Liouville in 1836. Of these, the first disappeared in 1832, but the others continue to be published today and maintain a great scientific reputation.



Liouville's journal from 1836.

The second half of the 19th century saw the birth of the national mathematics societies. The reason for their creation lay in the need to involve the growing number of mathematicians and their constant scientific production in a system that

allowed scientific communication, discussion and evaluation. For these purposes the academies were proving excessively restrictive and elitist.

The oldest mathematical society is the Moscow Mathematical Society, created in 1864; its journal, *Matematicheskii Sbornik*, has been published since 1866. This was followed by the London Mathematical Society, created, together with its journal, *Proceedings of the London Mathematical Society*, in 1865.

This society served as a model for those that followed: the Mathematical Society of France and the *Bulletin de la société mathématique de France*, created in 1872; the Edinburgh Mathematical Society of 1883, and the *Proceedings of the Edinburgh Mathematical Society*, of 1884; the New York Mathematical Society, which in 1888 was the predecessor of the American Mathematical Society created in 1894, and which published its famous journal, *Transactions of the American Mathematical Society*, for the first time in 1899. This trend also reached Spain; in 1911, the Spanish Mathematical Society was created.

A case which is worthy of an aside is that of the Circolo Matematico di Palermo, created, together with its journal, *Rendiconti del Circolo Matematico di Palermo*, in 1884. It was a peculiar scientific society set apart by the fact that most of its members were

THE FIRST MATHEMATICAL SOCIETIES AND THEIR JOURNALS

- Moscow Mathematical Society (1864) / *Matematicheskii Sbornik* (1866).
- London Mathematical Society (1865) / *Proceedings of the London Mathematical Society* (1865).
- Mathematical Society of France (1872) / *Bulletin de la Société Mathématique de France* (1872).
- Edinburgh Mathematical Society (1883) / *Proceedings of the Edinburgh Mathematical Society* (1884).
- Palermo Mathematical Circle (1884) / *Rendiconti del Circolo Matematico di Palermo* (1884).
- German Union of Mathematicians (1890) / *Mathematische Annalen* (1868).
- New York Mathematical Association (1888) / *Bulletin of New York Mathematical Society* (1891).
- American Mathematical Society (1894) / *Transactions of the American Mathematical Society* (1899).

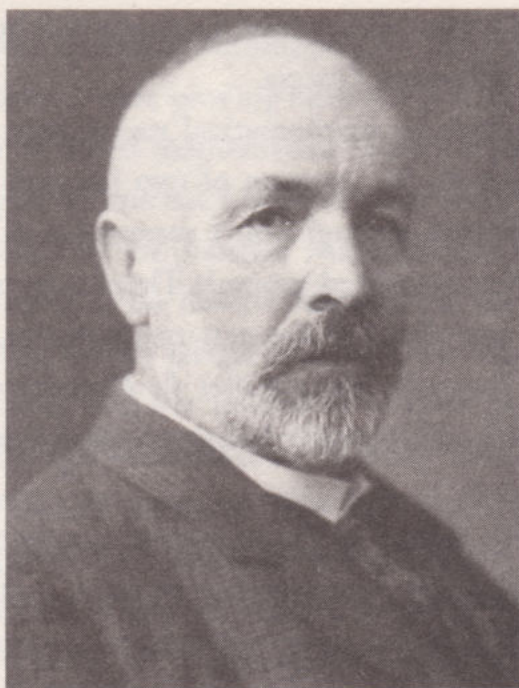
not Italians, among them the best mathematicians of the time. Later we will bump into its creator, the Sicilian mathematician Giovanni Guccia.

This panorama of new resources available to mathematical researchers was completed at the end of the 19th century with the creation of two journals that allowed mathematical literature to be followed through short summaries of the content of longer articles published in more specialised journals. There were now so many it was becoming difficult to find out about all the new mathematics that was being created. They were the review journals: *Jahrbuch über die Fortschritte der Mathematik*, which was first published in 1868, and *Répertoire bibliographique des sciences mathématiques*, published from 1885.

The process that we have summarised, thanks to the academic positions at universities, the specialised journals, the national mathematical societies and the review journals, meant that by the end of the 19th century mathematical research was more structured, professional and international. Today, we would say it was globalised.

You may have noticed that there is something missing from this historical overview. We have mentioned the creation of the mathematical societies and their journals in all the countries which have contributed to modern mathematics except for one, and one of the great ones, at that: Germany. By the end of the 20th century, the German model and German mathematics had acquired a great hegemony in Europe. A close look at the case of Germany will help us to understand the final step necessary in the international structuring of mathematics.

The creation of the Union of German Mathematicians, the *Deutsche Mathematiker-Vereinigung*, did not happen until 1890 (even though one of the most important German mathematics research journals, *Mathematische Annalen*, was created in 1868). The spirit that led to its creation came from the German mathematician Georg Cantor (born in Saint Petersburg). His investigations on the Fourier series led Cantor to the creation of modern set theory. This theory and its consequences were not accepted by important and influential German mathematicians (led by Leopold Kronecker of the University of Berlin) who, instead of seeing a tool for proposing and resolving new problems saw a threat to the solidity of new theorems. Cantor searched for a forum where he could express and defend his theories freely and uninhibited. He found it at the Union of German Mathematicians. He had the help of another great German mathematician, Felix Klein, whose interests included disseminating a new learning, research and academic model for mathematics.

*Georg Cantor.**Felix Klein.*

After the creation of the Union of German Mathematicians, Cantor dedicated himself to promoting the idea of an international mathematics congress. His objectives were the same as when he had been involved with the mathematics society and formed part of his personal struggle in the name of scientific freedom. (A famous phrase sums up one of the most profound characteristics of mathematics was his: "The essence of mathematics is in its freedom.") He was accompanied on this endeavour by important mathematicians who supported his idea enthusiastically. Among them were the Frenchmen Charles Hermite and Henri Poincaré.

An initial trial for an international congress was held in 1893 in Chicago, the city that hosted the World's Columbian Exposition the same year. Mathematicians at the recently created University of Chicago (among them two German students of Felix Klein) organised a five-day meeting in which they presented 44 scientific communiqués signed by mathematicians from seven different countries (Germany, USA, France, Italy, Austria, Switzerland and Russia) and including the most important mathematicians of the time. Nearly all missives were presented in the absence of their authors, as of the 45 participants only four came from outside of the United States. Felix Klein attended the congress as the Imperial Commissioner of the Prussian Ministry of Culture, and gave the opening lecture, where he championed the need for an international mathematics conference.



View of the World's Columbian Exposition in Chicago, held in 1893.

First date on neutral ground

Various proposals about possible hosts for the first international congress did the rounds among mathematicians towards the end of the 19th century, including Kazan (in Russia), Belgium, Paris and others. In the end the most impartial option in a Europe deeply affected by national rivalry triumphed: Zurich, in always neutral Switzerland.

It has never been easy to organise an international event, and even less so when it is the first one. Twenty-one mathematicians from nine countries set about the task in January 1897, sending a letter to 2,000 mathematicians around the world, inviting them to take part. The invitation was not sent directly by mail, but was distributed through mathematicians in 12 states (Germany, the Austro-Hungarian Empire, Belgium, USA, France, Great Britain, Greece, Holland, Italy, Portugal, Russia and Sweden).

The date was set for 9–11 August 1897 at the Federal Polytechnic Institute in Zurich (a prestigious scientific institution currently known as the ETH). This first congress sowed the seeds for what have come to be the international congresses of mathematicians, so we will pause to look at a few details of its progress and conclusions.

In order to evaluate the first international congress, we should take into account the fact that the vast majority of the mathematical researchers at the time did not

know each other personally. They had read articles by one another and, at most, had exchanged letters with scientific results. This is why the organisers prepared a welcome party in the famous Tonhalle Concert Hall in Zurich the night before the opening of the congress, after the arrival of the attendees. The minutes of the congress tell us that while “thoroughly enjoying the stimulating conversation and the joyous sound of glasses,” the participants made the lively meeting last until midnight. Adolf Hurwitz greeted those present on behalf of the organisers:

“May the inspiring force of personal communication rise during these days, providing plenty of occasions for scientific discussions. May we together enjoy the relaxed and cheerful comradeship, enhanced by the feeling that here representatives of many different countries feel united by the most ideal interests in peace and friendship.”

These words have become a symbol of the value of international mathematical cooperation and have been a guide for the international congresses that followed.



Adolf Hurwitz.

The first task was to decide how to organise a congress with these characteristics, since there was clearly a lack of experience in doing so. The plan of the scientific

programme arranged for the occasion has lasted and the same basic plan is used today. On the one hand there were plenary sessions, which covered subjects of general interest and whose lectures were given by invited speakers to all the participants. At the Zurich congress there were four plenary conferences:

- ‘Sur les rapports de l’analyse pure et de la physique mathématique’, by Henri Poincaré from Paris.
- ‘Über die Entwicklung der allgemeinen Theorie der analytischen Funktionen in neuerer Zeit’, by Adolf Hurwitz from Zurich
- ‘Logica matematica’, by Giuseppe Peano from Turin.
- ‘Zur Frage des höheren mathematischen Unterrichtes’, by Felix Klein from Göttingen.

On the other hand, the work of the congress was divided into specific sections, in which shorter presentations were presented, grouped according to common themes. There were five sections at the Zurich congress:

- I. Arithmetic and algebra.
- II. Analysis and theory of functions.
- III. Geometry.
- IV. Mathematical mechanics and physics.
- V. History and bibliography.

This division of mathematics shows the areas that were being developed at the time. Among the papers presented in the sections were, for example, one by the young Belgian mathematician Charles de la Vallée Poussin on the subject of prime numbers.

The congress dealt with the issue of national balance, which has been one of the great dangers and obsessions in Europe’s history, with tact and care. There was a reason the chosen host was Switzerland, “at the intersection of the long railways that run from Paris to Vienna and from Berlin to Rome”, as attendees were reminded at the opening ceremony. The national balance was also evident in terms of languages, as it was decided that the official tongues of the congress were German and French, and that the minutes of the congress would be drafted in both languages. The use of English and Italian was also allowed at the lectures and public speeches. The same national balance was apparent in the choice of plenary speakers: a German, a

Frenchman, an Italian and a Swiss (Hurwitz, a German based in Switzerland). During the congress there was a space for the history of mathematics in which tribute was paid to the past glories of mathematics. Thus, a long and flourishing eulogy

THE PRIME NUMBER THEOREM

We owe the first proof of the infiniteness of prime numbers to Euclid. But how infinite are the prime numbers? We call the number of primes less than or equal to N , $\Pi(N)$. In the 18th century, the French mathematician Adrien-Marie Legendre and the German Carl F. Gauss conjectured (when Gauss was 15 years old) a relation between $\Pi(N)$ and the quotient $N/\log N$. In 1852, the Russian mathematician Pafnuty Lvovich Tchebyshev proved that $\Pi(N)$ must be comparable to $N/\log N$. It was in 1896 when the French mathematician Jacques Hadamard and the Belgian mathematician Charles de la Vallée Poussin proved, simultaneously and independently, that, as N grows, $\Pi(N)/(N/\log N)$ tends towards 1.

Sur la théorie des nombres premiers.

Par

CH. DE LA VALLÉE POUSSIN à Louvain.

M. de la Vallée Poussin s'est occupé de la fréquence des nombres premiers de différentes formes dans un Mémoire étendu, publié dans les *Annales de la Société scientifique de Bruxelles* (1896) sous le titre: *Recherches analytiques sur la théorie des nombres premiers*.

Voici quelques conclusions de ce travail, concernant les nombres premiers d'une forme linéaire primitive $Mx + N$:

1° L'expression

$$\frac{\varphi(M)}{y} \sum_{q_N < y} lq_N,$$

dans laquelle la somme est étendue aux nombres premiers $< y$ et de la forme $Mx + N$, a pour limite l'unité quand y tend vers l'infini.

2° La différence

$$\varphi(M) \sum_{q_N < y} \frac{lq_N}{q_N} - ly$$

tend vers une limite finie et déterminée quand y tend vers l'infini.

3° Le nombre des nombres premiers de la forme $Mx + N$ et $< y$ peut se représenter par l'expression

$$\frac{1+s}{\varphi(M)} \frac{y}{ly}$$

où s tend vers zéro quand y tend vers l'infini.

On a des conclusions analogues concernant les nombres premiers représentables par une forme quadratique de déterminant négatif ($-D$).

*De la Vallée Poussin's communication
from the 1897 minutes.*

was dedicated to the great Swiss mathematicians: the brothers Jakob, Johann and Daniel Bernoulli, Leonhard Euler and Jakob Steiner. and the need to publish the complete works of Leonhard Euler was debated (there were several important editors among the attendees: Alfred Ackermann-Teubner of Leipzig, Albert Gauthier-Villars of Paris and Ulrico Hoepli of Milan). The great mathematicians of the time were also honoured: a toast was made to the general honour of the great French mathematician Charles Hermite and he was sent a telegram.

There was a feeling that mathematics was entering a new era based on international cooperation. This led to the creation of a list of tasks on which it was necessary to work by international agreement. Among them were the unification of mathematical terminology and units; accelerating information on publications (the review journals were very slow); creating a classification of mathematics which allowed bibliographical work (this came from the speed at which mathematical results were being produced); creating an international directory of mathematicians with all of their addresses and fields of speciality (this idea had to wait until 1958, the year in which the first International Directory of Mathematicians was created); even a biographical dictionary of mathematicians including photographs of the most eminent ones was suggested.

The scientific journals of the time, including *Science*, told their readers of the success of the congress. French mathematician and president of the Mathematical Society of France, Émile Picard, stated at the farewell banquet that “the success of our first congress is a guarantee for the future of the recently founded institution”.

The success was two-fold. Firstly, this was so because of attendance and scientific achievement: 208 mathematicians from 16 countries attended. There were four plenary conferences and 30 presentations in the sections. Many of the period's eminent mathematicians participated, including, as well as those mentioned above, Fredholm, Hausdorff, Lindelöf, Mittag-Leffler, Minkowski, Schönflies, Volterra and many more.

But not everybody supported the congress: not one mathematician from the University of Berlin attended. In a certain sense, this first international congress was a congress of the mathematicians of Göttingen, led by Felix Klein and Hilbert, carrying out Cantor's plans.

The other success story of the congress was the atmosphere in which it was conducted. This can be seen clearly in the ‘Congress Regulations’ that were approved, where it was established that the first two objectives of the international congresses of mathematicians were:

- a) To promote personal relations between mathematicians of different countries.
- b) To show the current status of the different areas of mathematical sciences and their applications, and discuss problems of particular importance.

Nach Ländern geordnet ergibt diese Teilnehmerliste die folgende Gruppierung:

Land.	Herren.	Damen.
Schweiz	60	8
Deutschland	41	12
Frankreich	23	6
Italien	20	5
Oesterreich-Ungarn . . .	17	3
Russland	12	1
Nordamerika	6	1
Schweden	6	—
Finland	4	1
Belgien	3	—
Dänemark	3	—
Großbritannien	3	—
Holland	3	—
Spanien	1	1
Griechenland	1	—
Portugal	1	—
	204	38

Im ganzen waren also 16 Länder durch 242 Teilnehmer vertreten, worunter 38 Damen.

Participants by country in 1897.

We have already mentioned that the order of these two objectives demonstrates the value given to personal relations between mathematicians as the engine of mathematical development. The enjoyment shown during the social activities reflects that desire to develop personal relations. Among the various excursions and banquets organised for the participants, which gave the congress a wonderful atmosphere of camaraderie, the steamboat trip through Lake Zurich to the beautiful village of Rapperswil was particularly memorable. On the return journey Veltheimer and Regensberger wines from the cellars of Zurich were served, and upon their return the participants found the city illuminated in their honour.

The future of the international congresses was discussed in the closing session of the congress. A decision was made that it would take place at intervals of three and five years and that at the end of each congress the host for the next one would be assigned. It was agreed that the next one would be in Paris in 1900, organised by the Mathematical Society of France. This is where the only glimmer

of national rivalry was seen: Felix Klein, president of the Union of German Mathematicians at the time, immediately declared his society's interest in organising the 1904 congress.



Lithography of the 1897 congress in Zurich.

Hilbert's lecture

“Who among us would not be happy to lift the veil behind which is hidden the future; to gaze at the coming developments of our science and at the secrets of its development in the centuries to come? What will be the goals toward which the spirit of future generations of mathematicians will head? What methods, what new facts will the new century reveal in the vast and rich field of mathematical thought?

“History teaches the continuity of the development of science. We know that every age has its own problems, which the following age either solves or casts aside as profitless and replaces by new ones. If we could have an idea of the probable development of mathematical knowledge in the immediate future, we must let the unresolved questions pass before our minds and look over the problems that the science of today sets and whose solution we expect from the future. Lying at the meeting of the centuries, today seems to me a good day to review these problems. For the close of a great epoch not only invites us to look back into the past but also directs our thoughts to the unknown future.”

The congress in Paris will always be remembered for Hilbert's conference. As we can see by the way in which it started, it was an unusual event. In it Hilbert did not present the solution to any problem. Instead, he proposed a list of problems whose solutions he believed would determine or illuminate mathematics in the 20th century. In the long preamble to the conference, Hilbert spoke of the nature of mathematical problems and their role in the development of science. His declaration of faith in the development of mathematics is very well known:

“For the mathematician there is no *ignorabimus*.”

This was a response to the tendency of some to defend a lack of knowledge. German philosopher Emil du Bois-Reymond stated “*Ignomarus et ignorabimus*”, meaning we do not know and we will not know. In contrast, Hilbert declared his conviction that, in mathematics, everything can be explained.

Hilbert's lecture had the German title ‘*Mathematische Probleme*’, although a summary printed in French was entitled: ‘*Sur les problèmes futurs des mathématiques*’. In it Hilbert presented a list of 23 problems, of which, due to time restraints, he only discussed ten. The fame of these problems has grown with time, lending prestige to any contribution that helped with their solution. It is paradoxical that Hilbert's lecture was not a plenary conference. It was given in the History and Bibliography section. The reason is simple: Hilbert took a long time to prepare the lecture, discussing its content with his colleagues Hermann Minkowski and Adolf Hurwitz, such that when it was ready the programme for the congress was already finalised.

The problems were:

- I. Cantor's problem on the continuum potential.
- II. On the compatibility of the arithmetical axioms.
- III. On the equal volume of two tetrahedra with equal bases and heights.
- IV. The straight line problem, the shortest route from one point to another.
- V. On the notion of continuous groups of Lie transformations, abstracting from the hypothesis that the functions which define the groups will be susceptible to differentiation.
- VI. The mathematical treatment of the axioms of physics.
- VII. The irrationality and transcendence of certain numbers.
- VIII. Problems on prime numbers.
- IX. Demonstration of the law of the reciprocity theorem in any body of numbers.
- X. On the possibility of resolving a Diophantine equation.
- XI. On quadratic forms with any algebraic coefficients.
- XII. Extension of the Kronecker theorem on abelian bodies to any domain of algebraic rationality.
- XIII. The impossibility of the resolution of the seventh degree equation by means of functions with only two parameters.
- XIV. Demonstrating that certain systems of functions are finite.
- XV. Rigorous foundation of Schubert's enumerative geometry.
- XVI. Problems of the topology of algebraic curves and surfaces.
- XVII. Representation of the shapes defined by means of the sum of squares.
- XVIII. Partition of space into congruent polyhedra.
- XIX. Are the solutions of regular problems in the calculation of variations always analytic?
- XX. The Dirichlet problem in the general case.
- XXI. Demonstration of the existence of linear equations with a monodromy group assigned.
- XXII. Uniform expression of analytical relations by means of automorphic functions
- XXIII. Extension of the methods of the calculus of variations.

Hilbert discussed the ten problems at the conference and, later, presented all 23 problems in an article.

SUR LES

PROBLÈMES FUTURS DES MATHÉMATIQUES,

PAR M. DAVID HILBERT (Göttingen),

- I. — Problème de M. Cantor relatif à la puissance du continu.
- II. — De la non-contradiction des axiomes de l'Arithmétique.
- III. — De l'égalité en volume de deux tétraèdres de bases et de hauteurs égales.
- IV. — Problème de la ligne droite, plus court chemin d'un point à un autre.
- V. — De la notion des groupes continus de transformations de Lie, en faisant abstraction de l'hypothèse que les fonctions définissant les groupes sont susceptibles de différenciation.
- VI. — Le traitement mathématique des axiomes de la Physique.
- VII. — Irrationalité et transcendance de certains nombres.
- VIII. — Problèmes sur les nombres premiers.
- IX. — Démonstration de la loi de réciprocité la plus générale dans un corps de nombres quelconque.
- X. — De la possibilité de résoudre une équation de Diophante.
- XI. — Des formes quadratiques à coefficients algébriques quelconques.
- XII. — Extension du théorème de Kronecker sur les corps abéliens à un domaine de rationalité algébrique quelconque.
- XIII. — Impossibilité de la résolution de l'équation générale du septième degré au moyen de fonctions de deux arguments seulement.
- XIV. — Démontrer que certains systèmes de fonctions sont finis.
- XV. — Établissement rigoureux de la Géométrie énumérative de Schubert.
- XVI. — Problèmes de topologie des courbes et des surfaces algébriques.
- XVII. — Représentation des formes définies par des sommes de carrés.
- XVIII. — Partition de l'espace en polyèdres congruents.
- XIX. — Les solutions des problèmes réguliers du calcul des variations sont-elles nécessairement analytiques?
- XX. — Problème de Dirichlet dans le cas général.
- XXI. — Démonstration de l'existence d'équations différentielles linéaires ayant un groupe de monodromie assigné.
- XXII. — Relations analytiques exprimées d'une manière uniforme au moyen de fonctions automorphes.
- XXIII. — Extension des méthodes du Calcul des variations.

Hilbert's 23 problems in the 1900 minutes.

HILBERT'S PENDING PROBLEMS

Not all the problems proposed by Hilbert are of the same nature. Some of them are precise and specific, and others are proposed programmes for research. In general, most of the problems that are susceptible to being solved (as some others are somewhat non-specific in their wording), are considered to be totally, partially or substantially solved. There is not a complete consensus on which have not been resolved due to different interpretations of the meanings of the questions but, in short, numbers VIII and XVI are unresolved. The first of these problems, number VIII, is discussed in the final chapter.

This second international congress coincided with the celebration of the World Exposition in Paris at the turn of the century, which brought more than two hundred scientific congresses to the city. Despite the initial high expectations (more than one thousand mathematicians pre-registered), the final attendance was 250 participants from 26 countries, just 25 percent more than in Zurich. The cause was possibly the alarming information, which even appeared in the scientific journals, regarding overcrowding and extortion of visitors to the World Exposition.



A view of the World Exposition held in Paris in 1900.

The programme followed the template of the one designed in Zurich. There were four plenary conferences:

- ‘Sur l’Historiographie des Mathématiques’, by Moritz Cantor from Heidelberg.
- ‘Betti, Brioschi, Casorati, trois analystes italiens et trois manières d’envisager les questions d’analyse’, by Vito Volterra from Rome.
- ‘Sur le rôle de l’intuition et de la logique en Mathématiques’, by Henri Poincaré from Paris.
- ‘Une page de la vie de Weierstrass’, by Gösta Mittag-Leffler from Stockholm.

The same sections as those at the Zurich congress were maintained, and to these a new one was added on “Teaching and methods”. We will see how this concern for teaching would take on more emphasis in the following congresses.

A UNIVERSAL LANGUAGE FOR SCIENCE

The 1900 congress talked over the proposal of “adopting a universal language for science and trade”, following the model of Esperanto, which had recently been created. The proposal was not very well received, but the underlying problem was considered a serious one. As the Russian mathematician Alexander Vassilief showed, in the early 19th century three languages were sufficient for a scholar, Latin, English and French, but he warned that the existence of twenty or thirty scientific languages would be a great danger to the science. It was agreed “to propose to the scientific academies and societies to study the means necessary for combating the problems derived from the growing diversity of languages employed in scientific literature”.

The closing ceremony was held in the magnificent Richelieu amphitheatre at the Sorbonne University and was presided over by Poincaré. It was agreed that the Union of German Mathematicians would organise the following congress in 1904. The second congress had been a success, maintaining the continuity of the international congresses, with the highlight being Hilbert’s list of 23 problems.

LUXURY AND EXTRAVAGANCE IN PARIS IN 1900

Prince Roland Bonaparte, Napoleon’s great nephew, who presided over the history section of the international congress of mathematicians and was known for his patronage of scientific endeavours, organised a party at his palace which combined honouring the Shah of Persia, who was visiting Paris, with a scientific soirée for the participants of the congress. It was an extravagant affair, according to those who attended.

The congress of the German empire

In 1904, the German Empire was at its economic, military, social, cultural and scientific peak. In line with this, the international congress of mathematicians was held in Heidelberg on an epic scale. It was not in vain that it had significant subsidies from the imperial government, from the Kaiser and from the Archduke of Baden. This allowed a great publicity drive (in 25,000 copies of the world’s main mathematical research journals), which paid off: there were 336 participants from 20 countries, compared to 208 in Zurich and 250 in Paris.

Nach Ländern geordnet ergibt diese Teilnehmerliste die folgende Gruppierung:

Land	Hauptkarten	Nebenkarten
Deutsches Reich	173	31
Rußland	30	4
Österreich-Ungarn	25	3
Frankreich	24	5
Vereinigte Staaten von Nordamerika	15	4
Dänemark	13	8
Italien	12	2
Schweiz	12	1
Schweden und Norwegen	8	1
Großbritannien	7	1
Niederlande	6	—
Belgien	2	—
Japan	2	—
Rumänien	2	—
Argentinien	1	—
Bulgarien	1	—
Canada	1	—
Griechenland	1	—
Spanien	1	—
	336	60

Im ganzen waren also 19 Länder durch 396 Personen vertreten.

Participants by country in 1904.

The scientific programme matched the model of the Zurich congress: four plenary conferences, one in each of the main languages of mathematics, German, English, French and Italian:

- ‘The Mathematical Theory of the Top (Considered Historically)’, by Alfred G. Greenhill from London.
- ‘Le problème moderne de l’intégration des équations différentielles’, by Paul Painlevé from Paris.
- ‘La geometria d’oggi e i suoi legami coll’analisi’, by Corrado Segre from Turin.
- ‘Riemanns Vorlesungen über die hypergeometrische Reihe und ihre Bedeutung’, by Wilhelm Wirtinger from Vienna.

The number of lectures presented in the sections increased compared to previous congresses – to 78 – still within the usual sections, whose only substantial variations were the change of the name of section VI to ‘Pedagogy’ and that of section IV to ‘Applied mathematics’, previously called ‘Mathematical mechanics and physics’.

On the morning of 10 August, the congress witnessed a scene of great drama. A lecture was planned on “The hypothesis of the continuum”, given by Hungarian mathematician Julius König, in which the resolution of the first of the problems proposed by Hilbert in the Paris congress was announced. The problem involved a conjecture proposed by Cantor regarding the size of infinite sets. Interest was such that all the other lectures were cancelled so that the guests could attend. In the presence of Hilbert and Cantor, König demonstrated that Cantor’s conjecture was false. In the ensuing discussion, Cantor, in a state of intense excitement, publicly thanked God for having allowed him to live to see the refutation of his errors. News that something very important had happened reached the local newspapers, to the point where Felix Klein had to visit the Archduke to inform him of what had happened. Cantor’s fright did not last long: a few weeks later an indirect error was found in König’s proof. Cantor’s conjecture was not completely resolved until 1963.

The explicit appearance of applied mathematics is an example of one of the souls of mathematics that every so often flourishes at the international congresses. The Applied Mathematics section organised an activity that was hugely successful and very lively: the presentation by academic institutions and commercial companies of various “mathematical instruments and apparatus”. The following apparatus, among others, were presented:

- The ‘Triumphator’ calculator, by Bombicki and Lamm from Berlin.
- A campylograph and angle divider, by Chateau Frères, a mechanical precision company from Paris.
- The Coradi differentiator, the Abakanowicz integrator, the Payne–Coradi ‘parabolograph’ and a harmonic analyser by G. Coradi, from the Mathematical–Mechanical Institute in Zurich.
- A gyroscope (according to the Maxwell model) and a gyrostat (according to the Lord Kelvin model), from the Göttingen Mathematical Institute.
- The deformable Darboux hyperboloid, by A. Greenhill from London.
- The ‘Brunsviga’ and ‘Addograph’ calculators, by Grimme, Natalis & Co., from Brunswick.
- A cyclograph and an elipsograph, from the Technical School of Vienna.
- The ‘epidiaskop’ and the ‘episkop’, from optical workshop of Carl Zeiss in Jena.

The presentation was accompanied by demonstrations and speeches, which included a lecture on ‘The geometry of numbers’ by Hermann Minowski using the epidiaskop.

The only historical apparatus that was demonstrated was the calculating machine designed and built by Leibniz in 1674 (it had one small, but correctable, error – the operation $9,999 + 1$ gave 9,900), which was comparable to some of the calculators at the time. Only two copies of that beautiful machine have survived to this day; one is in Hanover and the other in Munich.

The historical remembrance section was, as always, emotional. There was a special mention of mathematicians who had passed away in recent years: Arthur Cayley in 1895, Francesco Brioschi in 1897, James Sylvester in 1897, Karl Weierstrass in 1897, Sophus Lie in 1899, Erwin Christoffel in 1900, Charles Hermite in 1901, Lazarus Fuchs in 1902, Luigi Cremona in 1903 and George Salmon in 1904.

The main historical tribute was dedicated to the commemoration of the centenary of the birth of the great German mathematician Carl Gustav Jacob Jacobi (1804–1851) with the first plenary conference dedicated to his memory. This was attended by his family, and a scientific biography was distributed to all the participants.

In terms of publications, the importance of publishing a complete edition of Euler’s work was again emphasised, supporting the efforts of the Carnegie Institution and the academies of Berlin and Saint Petersburg in that regard. Felix Klein presented the first volume of an ambitious project for an encyclopaedia of all mathematical

MATHEMATICIANS' GRAVES

The mathematician Hermann Amandus Schwarz informed the congress about the situation with Jacobi's grave. As Jacobi's widow was unable to take responsibility for its upkeep, the Prussian Academy of Sciences had bought the property at the Trinity Church in Berlin and had dignified it by installing a fence. The congress was shown photographs of the final condition of the grave.

knowledge, the *Encyklopädie der mathematischen Wissenschaften*. In turn, the French mathematician Jules Mol presented the French version of the encyclopaedia, clarifying (the Franco-German rivalry once again raising its ugly head) that it was not a mere translation of the German one; instead, many articles had been rewritten by French mathematicians. The publishers B.G. Teubner from Leipzig and Gauthier-Villars from Paris, who supported these scientific endeavours and whose directors attended the congress, were thanked.

The resounding memory of the congress is the magnificent hospitality with which the attendees were treated. An example of this is the excursion by boat on the Neckar River. Upon its arrival in Heidelberg the delegates were greeted by the bridges and castle decorated with lights and flairs as fireworks depicted Pythagoras' theorem in the sky.

The congress's success was recognised at the closing ceremony, and an invitation presented by the Italian mathematician Vito Volterra, from the mathematics section of the Accademia dei Lincei and the Circolo Matematico di Palermo, to hold the next congress in Rome in 1908 was accepted.

Lynxes and Sicilians

"Let us remember the Italy of the urban republics and the Renaissance, with names such as Fibonacci, Tartaglia, del Ferro, Ferrari and so many more, who prepared the historical maturity of the new spiritual and social demands of the blossoming of science. After Fibonacci, two movements appeared, one in the studies of pure theory and the other founded on studies applied to trade, in which Italy found renewed fortune. From here came Luca Paciolo's double-entry accounting and his flourishing school of commercial arithmetic."

This is how, during the opening ceremony of the international congress, held in one of the beautiful halls of the Capitolio, the Italian Minister of Public Instruction summed up Italy's rich history in the application of mathematics, to which the interest in studying problems based on physics was added in the mid-19th century. This scientific tradition combined perfectly with the Italian government's interest in promoting the application of mathematics, which led to it supporting the congress not only through the Ministry of Public Instruction, but also the ministries of agriculture, industry and commerce, finance and development. Four Italian insurance companies even provided financial support for the congress.

The international congress held in Rome in 1908 went down in history for its emphasis on the application of mathematics, but also, as we will see later, for the special attention it dedicated to mathematical education.

The bias towards applied mathematics and applications of mathematics can be appreciated throughout the entire congress, from the simple fact that it was presided by physicist Pietro Blaserna to the inclusion of three highly applied plenary conferences. Plenary sessions had now increased in number from four to ten.

- 'Le partage de l'énergie entre la matière pondérable et l'éther', by Hendrik Antoon Lorentz from Leiden.
- 'La théorie du mouvement de la lune; son histoire et son état actuel', by Simon Newcomb from Washington.
- 'Sur les trajectoires des corpuscules électrisés dans le champs d'un aimant élémentaire avec applications aux aurores boréales', by Carl Størmer from Kristiania.

Lorentz had received the Nobel Prize for physics in 1902, and Newcomb was a renowned US astronomer. The emphasis on applied mathematics was also apparent in the individual sections, where it was expanded to include mechanics, physics, geodesics, actuarial mathematics and several other applications. A lecture was given on the driving of cars and the balancing of boats.

Among the diverse subjects that were debated by the congress in Rome, programmes and methods used for teaching mathematics in secondary education is noteworthy. There was a consensus on its great importance. Reports were presented on the situation in several countries (Germany, Austria, Greece, Hungary, England, Italy, Spain and the USA). At the suggestion of American David Eugene Smith, the

THE GUCCIA MEDAL

The first scientific award associated with an international congress appeared at this congress. It was a special offering from one of the organising institutes of the congress, the Circolo Matematico di Palermo. It consisted of a gold medal and three thousand lira in cash for the first innovative work on the theory of algebraic curves. The prize had been announced at the previous congress in Heidelberg in 1904 and the jury had been named as Poincaré, Max Noether from Erlangen and Corrado Segre from Turin. None of the work presented was considered worthy of the prize, but it was awarded for the work of Francesco Severi de Padua. The prize was short-lived to say the least – this was the first and only time it was awarded.

congress created the International Commission on Mathematical Instruction (initials ICMI), which was presided over by Felix Klein until his death. This institution has continued throughout the years, and celebrated its centenary in 2008. Proof of the importance given to it by the international mathematical community lies in the fact that it has been headed by eminent mathematicians such as Jacques Hadamard, Marshall H. Stone, André Lichnerowicz and Jean-Pierre Kahane. Since 1969 the ICME (International Congress on Mathematical Education), which brings together thousands of participants, has been organised every four years.

A UNION

The internationalist euphoria of the congress led to the proposal of studying the creation of an international association of mathematicians. The following congress, held in Cambridge in 1912, dropped the idea.

The congress in Rome continued the run of success with 535 mathematicians attending, 10 plenary conferences held and 127 lectures in the various strands (in Heidelberg in 1904 there were 336, 4 and 78 respectively). The spirit of continuity encouraged by the congresses is also evident in the fact that the money left over from the organisation of the Heidelberg congress was incorporated into the budget of the Rome congress.

Distribuzione dei Congressisti per Nazioni.

	CONGRESSISTI	PERSONE DI FAMIGLIA	TOTALE
Italia	190	28	218
Germania	120	54	174
Francia	68	29	92
Austria-Ungheria	51	23	74
Inghilterra	22	11	33
Stati Uniti (America)	16	11	27
Russia	19	6	25
Svizzera	10	2	18
Svezia	5	1	6
Romania	6	—	6
Spagna	5	—	5
Danimarca	3	2	5
Olanda	3	1	4
Norvegia	2	2	4
Belgio	4	—	4
Grecia	3	—	3
Tunisia	2	—	2
Bulgaria	1	—	1
Canadà	1	—	1
Egitto	1	—	1
Messico	1	—	1
Serbia	1	—	1
TOTALE	535	165	700

Participants by country in 1908.

The most memorable of the social events at the congress were the concert held in an amphitheatre on the Mausoleum of Augustus in Rome, and above all the train journey to the Villa Adriana in Tivoli, where aperitifs were served. The attendees then visited the cascades at the Villa d'Este (courtesy of Archduke Ferdinand of Austria, whose later assassination would lead to World War I).

SICILIAN TYPOGRAPHERS' STRIKE

The minutes from this congress form three large volumes with more than a thousand pages. The plan was for the printing to be carried out by Tipografia Matematica di Palermo, at the expense of the Circolo Matematico di Palermo. But a strike by the militant Sicilian typographers' guild prevented the printing, which eventually had to be carried out in Rome, in the print room of the other scientific institution responsible for organising the congress, the Accademia dei Lincei. This is a legendary and prestigious scientific institution, founded at the turn of the 17th century in Rome. It dedicated its time, and its acute vision (like that of a lynx, which is where its name Lincei comes from), to all fields of science. Galileo Galilei was one of its first members.

In the land of Darwin

The long dispute throughout the 17th and 18th centuries between Leibniz and Newton, and their respective followers, over the creation of infinitesimal calculus had the collateral effect of keeping Britain relatively isolated from the rest of the European continent in terms of mathematical evolution. An indirect consequence of that was that British mathematics was much more oriented towards its applications and its relation with other sciences than was continental mathematics. As had already happened in the international congress in Rome in 1908, this bias towards the applied impregnated the following congress held in Cambridge in 1912. The focus on natural sciences was evident, for example, in the fact that the honorary president of the congress was the physicist Lord Rayleigh, who had received the Nobel Prize in 1904 for the discovery of argon. The same is true of the plenary conferences, of which half were focused on applied subjects, which even today seem far removed from mathematics:

- 'Periodicities in the Solar System', by Ernest W. Brown from New Haven.
- 'The Principles of Instrumental Seismology', by Prince B. Galitzin from Saint Petersburg.
- 'The Dynamics of Radiation', by Sir Joseph Larmor from Cambridge.
- 'The Place of Mathematics in Engineering Practice', by Sir W.H. White.

Another paradigmatic example of the omnipresence of applications were the visits that were organised for the participants of the congress – first of all to the university's

astronomical observatory, and then to the Cambridge Scientific Instrument Company, which specialised in the manufacture of high-precision scientific instruments. The founder of this company (which continued to operate until 1968) was Horace Darwin, one of Charles Darwin's children. The naturalist had studied at Cambridge University. Another of Darwin's sons, George, who presided over the congress (and also the Royal Astronomical Society), made a peculiar reflection on pure and applied mathematics in his opening speech:

"I appeal for mercy to the applied mathematician and would ask you to consider in a kindly spirit the difficulties under which he labours. Although our methods may often be wanting in elegance and may do but little to satisfy that aesthetic sense of which I spoke before, they are honest attempts to unravel the secrets of the Universe in which we live."

THE MATHEMATICAL TRIPOS

The congress paid homage to the Mathematical Tripos. The Tripos were extremely difficult exams that consisted of hundreds of questions which had to be answered in continuous sessions lasting for more than a week. They were obligatory for all Cambridge students, and their results remained on the graduates' CVs for their entire lives. As the vice-chancellor of the university explained during the opening ceremony of the congress, "They represent the oldest example in Europe of a competitive examination with an order of merit." They appear to take their name from the three-legged stool on which the students originally took the exam in the 18th century.

The congress welcomed the Swiss Society of Natural Sciences' decision to begin publishing, "in magnificent style", the complete works of Leonhard Euler. We have seen that this was a recurring issue in the previous international congresses. The project had the support of the Union of German Mathematicians, which contributed one third of the funds, and the French Mathematical Society, which would buy 40 copies as long as the articles were edited in their original language. All the estimations on the magnitude of the endeavour came up short. Today, some one hundred years later, 72 volumes of scientific articles have been published and work continues on the ten volumes that are estimated to be needed for his scientific correspondence (see www.leonhard-euler.ch).

The 1912 congress in Cambridge was a high point for the international congresses since their beginnings in 1897. A total of 574 mathematicians from 28 countries took part, with a relatively high attendance from non-Europeans – 87. Also, the charm of the medieval buildings which housed the giants of science, the receptions in palaces, the organ concerts and the invitations to have tea in country estates captivated all attendees. There was a clear feeling that the international congresses had a promising future.

DISTRIBUTION OF MEMBERS ACCORDING TO NATIONALITY

	Members	Members of family	Total
Argentina	5	—	5
Austria	20	3	23
Belgium	5	—	5
Brazil	1	—	1
Bulgaria	1	—	1
Canada	5	—	5
Chili	1	—	1
Denmark	4	2	6
Egypt	2	—	2
France	39	6	45
Germany	53	17	70
Greece	4	1	5
Holland	9	1	10
Hungary	16	3	19
India	3	—	3
Italy	35	6	41
Japan	3	—	3
Mexico	2	—	2
Norway	3	1	4
Portugal	2	1	3
Roumania	4	1	5
Russia	30	10	40
Servia	1	—	1
Spain	25	2	27
Sweden	12	2	14
Switzerland	8	2	10
United Kingdom	221	49	270
United States of America	60	27	87
	<u>574</u>	<u>134</u>	<u>708</u>

Participants by country in 1912.

The Swedish mathematician Gösta Mittag-Leffler invited those present, on behalf of the Swiss Royal Academy of Sciences and editor of the journal *Acta*

Mathematica (the best mathematical research journal in the world), to attend the 1916 congress in Stockholm. The optimism of the moment and prospects for the future also led to the presentation of invitations to the 1920 congress, in Budapest, and to Athens in 1924.

MORE MATHEMATICIANS' GRAVES

A group of participants in the congress went to the nearby Mill Road cemetery to honour the memory of the British mathematician Arthur Cayley and placed a wreath on his grave. Cambridge University, moved by the gesture, agreed to have a silver crown made as a permanent memorial.

Chapter 3

Wartime Disasters

Many historians consider that the 20th century did not begin, and therefore the 19th century did not end, until the outbreak of World War I, originally called the Great War, in 1914. It resulted in a new and different world. Four great powers collapsed – the German, the Austro-Hungarian, Ottoman and Russian empires – and a host of new countries arose from their ashes, among them the Soviet Union, with its unprecedented social system. Never had there been so many, and such profound, changes in such a short span of time. The post-war changes were traumatic, as was the war itself. The cost of the war in human lives, both in the military and civilian losses and the casualties and severely injured, had a tremendous impact on social life.

Science was not immune to the maelstrom of the war and the hurricane of its consequences – hatred among them. Before the end of the struggle the Allied governments decided to intervene in science, the importance for economic development was increasingly evident. Hence, the International Research Council (abbreviated to IRC) was created, the objective of which was formally declared as promoting international scientific cooperation, but its barely hidden agenda sought to eliminate Germany's presence from many fields of science. Although there were scientists in management positions in the IRC, its members were not scientists or scientific associations or academies; they were countries – in other words, governments.

Some mathematicians had a relevant role in the IRC. Among them was Frenchman Émile Picard, who was the president until 1931, and the Italian Vito Volterra was vice president. But the IRC's interest in mathematics ran deeper. At the IRC's inaugural meeting in 1919, in Brussels, two important decisions regarding mathematics were made. During the war, as was to be expected, the international congresses were suspended; therefore, the congress planned for Stockholm in 1916 was not held. The IRC decided to abandon it, and the next international congress was to be held in 1920 in Strasbourg. The choice was not innocuous: Alsace and its capital, Strasbourg, had been disputed for a long time by Germany and France.

The second decision by the IRC was to draw up the statutes of the future International Mathematical Union (UMI in its French initials) for approval. The union was founded at the Strasbourg congress.

Times of revenge

Mathematically speaking, the 1920 congress in Strasbourg was like the previous ones: it had an honorary president, the octogenarian French mathematician Camille Jordan; nearly 80 papers were presented in the traditional scientific sections and important mathematicians held five plenary conferences:

- ‘Questions in Physical Indetermination’, by Sir Joseph Larmor from Cambridge.
- ‘Relations between the Theory of Numbers and Other Branches of Mathematics’, by Leonard Eugene Dickson from Chicago.
- ‘Sur les fonctions à variation bornée et les questions qui s’y rattachent’, by Charles de la Vallée-Poussin from Lovaina.
- ‘Sur l’enseignement de la physique mathématique et de quelques points d’analyse’, by Vito Volterra from Rome.
- ‘Sur les équations aux différences finies’, by Niels Erik Nørlund from Lund.

But, although the number of countries represented was practically the same as at the congress in Cambridge in 1912, something had changed: 200 mathematicians attended and, moreover, 80 of them were French. This was the lowest figure since the start of the series of international congresses, and it would be the lowest in its history. Had the war dampened the enthusiasm stirred up among mathematicians by previous congresses?

There is actually a simpler but sour explanation: the congress was held under conditions imposed by the IRC, which impeded scientists from the losing side from attending the international congresses – Germany, the Austro-Hungarian Empire, Bulgaria and Turkey. This meant vetoing a very substantial part of the scientific community, both in terms of renown and number. But, furthermore, as the general secretary of the congress and French mathematician Gabriel Koenigs explained, the congress was not open. In other words, the attendees were chosen and had received a personal invitation; anyone who had not been personally invited was not able to attend. On top of this, some mathematicians declined the invitation, such

as Englishman Godfrey Harold Hardy and the Swede Gösta Mittag-Leffler, who publicly opposed the exclusion policy and appealed for friendly relations between mathematicians of all countries to be restored. But in the 1920s these voices were still in the minority.

RÉCAPITULATION PAR NATIONALITÉS

	<i>Congressistes.</i>	<i>Membres des familles.</i>	<i>Total.</i>
Angleterre.....	9	2	11
République Argentine	6		6
Australie	1		1
Belgique.....	10	4	14
Brésil	1		1
Canada.....	1		1
Danemark.....	3	1	4
Égypte	1		1
Espagne	10		10
États-Unis.....	11	4	15
France.....	80	32	112
Grèce.....	6	1	7
Hollande.....	5	1	6
Indes.....	2		2
Italie.....	5	2	7
Japon	2		2
Mexique.....	2		2
Norvège.....	4	2	6
Philippines.....	1		1
Pologne.....	2		2
Portugal.....	3	3	6
Roumanie.....	6	1	7
Russie.....	1	1	2
Serbie.....	1		1
Suède.....	1		1
Suisse.....	14	3	17
Tchéco-Slovaquie	12		12
	200	57	257

Participants by country in 1920.

These circumstances gave the congress an intense post-war atmosphere with visits to war memorials and battlefields, exaltations of the Alsatian identity in lectures and concerts, and provisions and benefits for those who had been officials in the allied armies were plentiful.

The culminating moment of the congress, held on a near-war footing, was the closing speech by the president of the congress and of the IRC, Émile Picard, who stated:

“In respect to certain relations broken by the tragedy of these last years, our successors will determine if a sufficiently long lapse of time and a sincere repentance could allow them to resume some day, and if the ones who excluded themselves from the civilised nations deserve to re-enter again. For we, too close to the events, still assume the fine words said by Cardinal Mercier during the war: to pardon certain crimes is to become accomplice with them.”

In judging these harsh words we should take into account that Picard's own family died in the war.



Émile Picard.

The closing of the congress was different from what had become traditional, as proposals for the next congress were not presented or discussed. The reason is that a few days beforehand, in a restricted meeting, the International Mathematical Union had been founded, and in it the statutes created by the IRC were approved and the congresses of 1924 and 1928 were set for New York and Belgium, respectively. The attendees were simply informed of the decision which had been made.

These impositions were short-lived: as the time of the next congress approached, it became clear that the American Mathematical Society was not in a state to organise a

congress in which the exclusion policy of mathematicians from the old central powers was upheld. There were several reasons for this. Firstly, the lesser effects of the war on the US, which reduced the emotional toll and, secondly and more importantly, the strong scientific relationship between the American and German academic worlds, as the latter was fundamental in the development of the former. This made it practically impossible to achieve the financial support necessary for organising a congress based on a policy of excluding the Germans.

The continuity of the international congresses was seriously compromised, as the IRC and, therefore, the International Mathematical Union, were not prepared to allow an open congress in which German mathematicians were allowed to participate. The solution came *in extremis* at the hands of Canadian mathematician John Charles Fields, who offered to organise a congress in Toronto in 1924 respecting the exclusion policy.

Fields' immeasurable efforts bore fruit, and the congress was held in August 1924 with a large attendance: 444 participants from 28 countries, among which there were historical firsts, such as the Irish Free State (recently independent from the UK), Czechoslovakia (state created in 1918) and several Soviet republics (including Georgia, Russia and Ukraine). Although two thirds of the attendees were from North America, it was a great achievement to receive so many participants while maintaining the exclusion policy against Germans, Austrians, Hungarians and Bulgarians.

Following the English scientific tradition, the congress had a distinct 'applied' feeling to it, as can be seen in the new list of scientific sections at the congress, where subjects related to engineering and industry dominate:

- I. Algebra, theory of numbers and analysis.
- II. Geometry.
- III (a). Mechanics and physics.
- III (b). Astronomy and geophysics.
- IV (a). Electrical, mechanical, civil and mining engineering.
- IV (b). Aeronautics, naval architecture, ballistics and radiotelegraphy.
- V. Statistics, actuarial science and economy.
- VI. History, philosophy and didactics.

GEOGRAPHICAL DISTRIBUTION

	A	B	C
Argentina	2	—	1
Australia	—	1	—
Belgium	6	2	—
Canada	107	4	7
Cuba	—	1	—
Czechoslovakia	3	—	—
Denmark	3	1	—
Egypt	—	1	—
France	24	18	3
Georgia	1	—	—
Great Britain	58	13	20
Greece	—	1	—
Holland	4	—	2
Hong Kong	1	—	—
India	2	—	—
Irish Free State	2	—	1
Italy	11	3	1
Japan	—	1	—
Jugoslavia	1	—	—
Malta	1	—	1
Mexico	1	—	—
Norway	5	1	—
Peru	—	1	—
Poland	2	1	—
Portugal	2	1	—
Roumania	1	1	—
Russia	4	10	—
Samoa	1	—	—
Spain	3	1	—
Sweden	3	2	—
Switzerland	4	1	—
Ukraine	1	2	—
United States	191	15	64
	444	82	100

A. Delegates and Members present in Toronto.

B. Corresponding Members.

C. Relatives accompanying those listed under A.

The list of the participants by country at the Toronto congress in 1924.

The list of attendees also shows the applied and industrial nature of the congress, with participants from the companies Eastman Kodak, General Electric, American Telephone and Telegraph (AT&T), Marconi and also from the US and French ministries of war. As happened in Cambridge in 1912, the visit organised by the congress was also based on an interest in industrial and economic activity: a visit was made to the Queenston-Chippawa hydro-electric dam, on the Niagara River, which had been opened three years previously.

Scientifically speaking, the congress was impeccable, with 241 presentations and eight plenary conferences (where, strangely, there was very little applied mathematics):

— ‘La théorie des groupes et les recherches récentes de géometrie différentielle’,
by Élie Cartan from Paris.

- ‘Outline of the Theory to Date of the Arithmetics of Algebras’, by Leonard Eugene Dickson from Chicago.
- ‘Considérations sur une équation aux dérivées partielles de la physique mathématique’, by Jean Le Roux of Rennes.
- ‘Non-Euclidian Geometry from Non-projective Standpoint’, by James Pierpoint from New Haven.
- ‘Sulle operazioni funzionali lineari’, by Salvatore Pincherle from Bologna.
- ‘La géometrie algébrique’, by Francesco Severi from Rome.
- ‘Modern Norwegian Researches on the Aurora Borealis’, by Carl Størmer from Oslo.
- ‘Some Characteristics Features of Twentieth-Century Pure Mathematical Research’, by William H. Young from London.

With regards to the complementary activities, this congress will be remembered for a fabulous excursion organised for the participants (and for the members of the British Association for the Advancement of Science who were also meeting in Toronto), whose objective was to: “Discover in detail the physical characteristics of Canada and its natural resources.” On the night of 17 August a train left Toronto on an intercontinental journey to Victoria and Vancouver on the Pacific coast, returning on 3 September – a 17-day trip in total!

The congress largely managed to bypass memories of the war, which had ended six years before, although its shadow remained. The Belgian mathematician Charles de la Vallée-Poussin laid a crown of flowers at the monument to the soldiers from the University of Toronto who had fallen during the war, and in his speech he remembered the 1920 congress, alluding to “the liberation of science from the sacrilegious hands that for so long had used it for their criminal aims”.

But this does not reflect the general atmosphere of the congress. For many who attended in North America it was a sad surprise not to find German mathematicians there, which led to a great deal of disquiet and protest. The mathematical societies of the USA, Denmark, Britain, Holland, Italy, Norway and Sweden appealed for the end of the exclusion policy. This was something that the Francophone IRC was not ready to grant but, as a result, the invitation to hold the 1928 congress in Belgium, which would have been carried out under the same conditions as previous ones, was withdrawn. So, for the first time in the history of the national congresses, the closing ceremony passed without having decided where the next congress would be held.

Time for reconciliation

The international situation changed in 1925 with the signing of the Locarno Treaties, in which the European nations renounced resorting to war as a means for solving their differences (not a macabre joke, as much as it may sound like in hindsight). In 1926 Germany joined the League of Nations. This led the IRC to change its statutes and invite Germany to join. The complex internal situation in Germany (because of, among other things, the existence of several scientific academies) meant that the response was not immediate. At that time the International Mathematical Union decided to hold the next congress in Bologna, from where the president of the union, Italian mathematician Salvatore Pincherle, hailed. He decided that the congress would be open to all mathematicians in the world and, consequently, invitations were sent to all the mathematical societies, including Germany's.

Tensions then heightened everywhere. In Germany, the mathematician Ludwig Bieberbach (who would later be a fervent Nazi and promoter of the magazine *Deutsche Mathematik*, which would boast about not including articles by Jewish mathematicians) led a movement to block the German attendance in Bologna. Against him stood David Hilbert, who strongly defended the need to attend. For its part, the IRC decreed that 'free attendance' made the congress illegal. The final front was formed by a large number of mathematical societies who agreed that they would not attend if participation was not open. In an act of political virtuosity, Pincherle obtained sponsorship for the congress from the University of Bologna, instead of the International Mathematical Union, and maintained its open nature.



Salvatore Pincherle



Ludwig Bieberbach.

The test was passed with great success. The congress opened on 3 September, 1928, in Bologna with 836 participants from 36 countries; the attendance was more than 45% greater than the Cambridge congress in 1912. Apart from continued political declarations, this ended the exclusion policy in the field of international cooperation in mathematics.

ELENCO DEI CONGRESSISTI

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DISTRIBUZIONE DEI CONGRESSISTI PER NAZIONI

	Effettivi	Persone di Famiglia	Totale
Argentina	7	—	7
Austria	9	1	10
Belgio	10	4	14
Brasile	2	—	2
Bulgaria	5	—	5
Canadà	4	3	7
Cecoslovacchia	15	7	22
Danimarca	9	5	14
Egitto	2	—	2
Finlandia	2	—	2
Francia	56	35	91
Germania	76	30	106
Giappone	11	2	13
Grecia	8	2	10
Guatemala	1	2	3
India	5	2	7
Inghilterra	47	17	64
Irlanda	2	—	2
Italia	336	76	412
Jugoslavia	4	1	5
Lettonia	1	—	1
Lituania	1	1	2
Norvegia	8	4	12
Olanda	9	6	15
Palestina	6	—	6
Polonia	31	10	41
Romania	11	8	19
Russia	27	3	30
Serbia	1	—	1
Spagna	11	1	12
Stati Uniti d'America	52	24	76
Svezia	4	3	7
Svizzera	29	19	48
Turchia	2	2	4
Ucraina	10	3	13
Ungheria	22	9	31
	836	280	1116

Participants by country in 1928.

Symbolically, the end of the exclusion policy took place when David Hilbert, who embodied the spirit of international cooperation as few other mathematicians did, entered the congress hall to head the German delegation. There was a long

silence, the Germans had not participated in the congresses since 1912. Suddenly, loud applause broke out. The international mathematical community rejoiced over the fact that his desire to cooperate had eliminated political interference. Hilbert's words in those moments have become a universal symbol of mathematics:

"It makes me very happy that after a long, hard time all the mathematicians of the world are represented here. This is as it should be and as it must be for the prosperity of our beloved science.

"Let us consider that we as mathematicians stand on the highest pinnacle of the cultivation of the exact sciences. We have no other choice than to assume this highest place, because all limits, especially national ones, are contrary to the nature of mathematics. It is a complete misunderstanding of our science to construct differences according to peoples and races, and the reasons why this has been done are shabby ones.

"Mathematics knows no races (...). For mathematics, the whole world is a single country."

Regarding the mathematics, the congress had 16 plenary conferences (given by, among others, Hilbert, Borel, Castelnuovo, Fréchet, Hadamard, Lusin, Tonelli, Veblen, Volterra and the physicist Theodore von Karman) and 419 presentations in the scientific sections, where again there was an expansion of the applied sections. For the first time, the calculation of probabilities appeared among the contents of the sections, demonstrating the strength of an emerging branch. Just as in Toronto in 1924, there was a visit to a recently opened hydroelectric plant, located between Riva di Garda and Lago di Ledro.

Historical remembrance was given in the form of the presentation of a recently discovered chapter of *Algebra* by Italian renaissance mathematician Rafael Bombelli, visits to the Scipione del Ferro house including the laying of a commemorative plaque, to the Bonaventura Cavalieri church and to Ferrara, where Copernicus had studied canon law.

The closing ceremony, at the Palazzo Vecchio in Florence, accompanied by amusingly attired heralds and trumpeters, was by all accounts magnificent. The city mayor reminded participants of Tuscany's wonderful contributions to mathematics: Leonardo Fibonacci, Rafael Canacci, Leonardo da Vinci and Galileo Galilei and, just as in ancient Greece where art and science combined, so had Tuscany borne artists such as Botticelli and Rafael. The mayor also spoke of the region's libraries,

rich in treatises of arithmetic, algebra and geometry from the end of the Middle Ages, documents for the traders who were bankers to the popes, kings and emperors of Europe. A lecture from US mathematician George David Birkhoff on “Some mathematical elements of art” was followed by an exhibition of old mathematical manuscripts and books from the Florence National and Lorentian libraries.

The Bologna congress also served as an international exhibition of Benito Mussolini’s fascist state; in fact, Mussolini provided great support for the congress. Senior members of the National Fascist Party, the Casas del Fascio, the Fascist Unions, etc., were present throughout. In his welcoming speech, the mayor of Bologna trumpeted: “Fascist Bologna is proud to offer its hospitality and exhibit what it has become under the vivifying impulse of fascism.”

LANGUAGES OF THE CONGRESS

The official languages of the congress were Italian, French, German, English, Spanish, Classical Latin (in fact, the Dean of the University of Bologna gave his speech in Latin) and Interlengua or Latin *sine flexione*, a simplified Latin created by the Italian mathematician Giuseppe Peano.

Times of peace

The 1932 congresses in Zurich (for the second time) and in 1936 in Oslo took place in a state of internal concordance that was unheard of in previous years, despite the fact that the European political situation was becoming increasingly strained. The impact of the terrible economic crisis suffered in Europe during those years, caused by the Great Depression that began in 1929, was unavoidable. This was evident in both the speeches and the modest organisation of the congress. (The 1932 concert at the Tonhalle in homage to the attendees was only possible thanks to anonymous donations from certain ‘music lovers’ from Zurich.)

Anything related to the controversy caused by the exclusion policy was treated with great tact. In Zurich, in 1932, Ludwig Bieberbach was invited as a plenary speaker (his mathematical nous was more than sufficient for the job) and a telegram was sent to Émile Picard as a sign of admiration and respect, while David Hilbert was honoured once more, the congress all standing as he entered the hall.

Verteilung der Kongressmitglieder auf die Staaten

		Teilnehmer Begleiter				Teilnehmer Begleiter	
				Übertrag 353		87	
Ägypten	5	2		Lettland	1	1	
Afrika	2	1		Mexiko	1	—	
Belgien	7	1		Norwegen	9	5	
Bulgarien	3	1		Österreich	10	—	
Canada	2	—		Palästina	2	—	
China	3	3		Persien	1	—	
Dänemark	6	2		Polen	20	2	
Deutschland	118	24		Portugal	2	1	
England	37	12		Rumänien	7	5	
Finnland	3	—		Russland	10	2	
Frankreich	69	20		Schweden	5	2	
Griechenland	5	—		Schweiz	144	41	
Holland	16	1		Spanien	10	—	
Japan	3	1		Tschechoslowakei	12	1	
Indien	2	—		Türkei	2	—	
Irland	4	2		Ungarn	12	3	
Italien	64	17		U. S. A.	66	36	
Jugoslawien	4	—					
Übertrag 353		87		Total		667	186

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RÉPARTITION DES MEMBRES DU CONGRÈS
PAR DIFFÉRENTS PAYS

		Personnes				Personnes	
		Participants: adhérentes:				Participants: adhérentes:	
				Transporté 276		107	
L'Union Sud-Africaine	2	1		Hongrie	5	1	
Algérie	1	—		Inde	3	—	
Allemagne	35	2		Iran	2	1	
Australie	1	—		Irlande	6	3	
Autriche	10	2		Islande	1	—	
Belgique	9	3		Italie	5	2	
Bulgarie	2	—		Japon	4	—	
Canada	7	6		Lettonie	3	1	
Chine	1	—		Norvège	59	25	
Danemark	22	5		Pays-Bas	15	4	
Égypte	3	—		Palestine	5	1	
Espagne	8	—		Pologne	25	4	
Esthonie	1	—		Roumanie	9	6	
États-Unis	86	56		Suède	26	10	
Finlande	8	1		Suisse	20	11	
France	28	13		Tchécoslovaquie	10	3	
Grande-Bretagne	48	18		U. R. S. S.	11	—	
Grèce	4	—		Yougoslavie	2	3	
A transporter 276		107		Total 487		182	

List of the participants by country in 1932 (top) and in 1936.

The German mathematician Hermann Weyl aided the reconciliation by dealing with a lesser, but quite delicate subject in one of his speeches. The first congresses, from 1897 to 1912, had been numbered. So, the Cambridge congress in 1912 was the 5th International Congress of Mathematicians. The disruption caused by the Great War and the imposition of the exclusion policy meant that the Strasbourg congress in 1920 and the Toronto edition in 1924 were not open to all mathematicians. Therefore, part of the mathematical community did not consider them to be part of the series of international congresses. An example of this is that the minutes of the Bologna congress refer to the 6th International Congress of Mathematicians. Of course, defenders of the International Mathematical Union and the IRC, who were mainly French and Belgian, did not see it that way. Hermann Weyl explained that the current congress, in Zurich in 1932, was number N , where the only thing the axiomatic grounds allowed to be determined was that N was between 7 and 9 (7 if the Strasbourg and Toronto congresses are not accepted, and 9 if they are). The solution proposed by Weyl was accepted, and since then the congresses have not been numbered.

Signs of change can be seen in the scientific programmes at these two conferences. There was a large number of plenary conferences, which demonstrated the growing diversity of the mathematical community: 20 in Zurich (by Carathéodory, Julia, Carleman, Cartan, Bieberbach, Morse, Noether, Bohr, Severi, Nevanlinna, Riesz, Sierpinski, Bernstein, and quantum physicist Wolfgang Pauli, among others) and 19 in Oslo (by Cartan, Siegel, Veblen, Birkhoff, Ahlfors, Van der Corput, Banach, Fréchet and Wiener, among others). But it is on the list of scientific sections where the changes that were occurring in mathematics can be appreciated:

- I. Algebra and theory of numbers.
- II. Analysis.
- III. Geometry and topology.
- IV. Calculation of probabilities, mathematical statistics, actuarial mathematics and econometrics.
- V. Mathematical physics, astronomy and geophysics.
- VI. Rational and applied mathematics.
- VII. Logic, philosophy and history.
- VIII. Pedagogy.

We can see that there are new mathematical areas on the list, such as topology. In the same sense of progressive specialisation, the US mathematician Solomon

Lefschetz, of Princeton, informed the congress in Oslo in 1936 of the preparations, “if circumstances allow it”, for a congress on topology in Warsaw in 1939. It may have been one of the first congresses dedicated to a specific branch of mathematics, but it never took place due to the bellicose political situation.

In the historical remembrance section, the Oslo congress paid homage to the great Norwegian mathematician Sophus Lie and unveiled a large bust of him, paid for by a group of anonymous donors; the bust can still be seen today at the University of Oslo.

Although it did not reach the levels of the Bologna congress in 1928, participation was high: 667 in Zurich in 1932 and 487 in Oslo in 1936. In the latter congress, the political tensions running through Europe began to tell. It is interesting to note the change in participation of mathematicians from the Soviet Union. While there were 27 in 1928, in 1932 there were only 10, and in 1936, 11, but in the last case practically none of them, not even the invited plenary speakers, actually attended the congress – Stalin’s purges were in full flow. Another interesting case is that of Italy, with only five participants signed up in 1936, most of whom did not attend either. The reason was the Norwegian protest against the invasion of Abyssinia by Mussolini, which led to the Italian government prohibiting their citizens from attending the congress. The Oslo congress sent a message of support to Vito Volterra, who had been expelled from his university for not swearing allegiance to Mussolini’s regime.



Vito Volterra.

A significant event for mathematics occurred at the congresses of 1932 in Zurich and 1936 in Oslo – a medal that would become the most important decoration in all mathematics was instituted and awarded. During the opening session of the 1932 congress a proposal from Canadian mathematician John Charles Fields, who had organised the congress in Toronto in 1924, was presented suggesting a scientific prize for young mathematicians.

Fields had passed away a few months before, so the proposal was made by his colleague and friend John L. Synge. The matter of prizes has always been controversial in mathematics, as there have always been those who question their usefulness. It was decided that the problem would be considered throughout the congress. Given its international nature, the following was agreed at the closing ceremony:

“The International Congress of Mathematicians held in Zurich accepts with thanks the offer made by the late professor Fields of two medals to be awarded to two mathematicians at intervals of four years by the International Congresses.”

A committee of five mathematicians was named to award the medal at the following congress.

During the opening ceremony of the 1936 congress in Oslo, held in a university hall decorated with murals by the painter Edvard Munch and in the presence of Haakon VII of Norway, the first two medals were awarded to Lars V. Ahlfors and American Jesse Douglas. The Greek mathematician Constantin Carathéodory explained the reasons for the prize: Ahlfors’ work on meromorphic functions and Douglas’s work on resolving the ‘plateau problem’. At the congress’s closing ceremony the commission responsible for awarding the medals at the next congress was named.

Since then, the awarding of the medals (which we are still yet to name; we will do so in the next chapter) has become the most exciting part of the international congresses. Rumours of lists of candidates and winners circulate for months, and they are finally unveiled on the opening day of the congress.

The Oslo congress concluded with the invitation of the American Mathematical Society to hold the next congress in 1940 in the United States, possibly in New York or somewhere else on the Atlantic coast.



Lars V. Ahlfors (left) and Jesse Douglas, the first two mathematicians to receive the medal proposed by John Charles Fields.

Women at the congress

In the 40 years in which the first ten international congresses took place the participation of women was low, as one might expect.

In the first Zurich congress in 1897, there were four women among 208 participants; specifically, Dr. Fräulein Iginia Massarini from Rome, Professor Frau Vera von Schiff from Saint Petersburg, Professor Fräulein Charlotte Angas Scott (who was the first British woman to achieve a doctorate in mathematics and was vice president of the American Mathematical Society in 1905) from Pennsylvania and Dr. Frau Charlotte Wedell from Göttingen.

In Paris, in 1900, there were six female attendees out of a total of 250. They came from Berlin, Copenhagen, New York, Moscow, Pennsylvania and Saint Petersburg (the last two had also attended the 1897 congress). In Heidelberg, despite the fact that the attendance rose to 336, there were only two women, both of them Russians. In 1908, in Rome, of 535 only ten were women. The first presentation given by a woman at an international congress was scheduled to be given here by Dr. Laura Pisati (the title of the lecture was "Essay on a Synthetic Theory for Complex Variable Functions"). Unfortunately, eight days earlier she passed away.

The congress in Cambridge in 1912 marked a turning point. Thirty-eight women took part (out of a total of 574 participants), 30 of whom were British, and Miss H.P. Hudson presented the paper "On Binodes and Nodal Curves", the first given by a woman at an international conference. In 1928, in Bologna, there were 72 women out of a total of 836 participants.

At the Zurich conference of 1932, the German mathematician Emmy Noether gave the first plenary lecture by a woman, "Hyperkomplexe Größen und Darstellungstheorie in arithmetischer Auffassung". Fifty-eight years passed until the next one, which took place in 1990 in Kyoto – a long time and a very long journey. It should be noted that the International Federation of University Women and the American Association of University Women sent official representatives to the 1932 congress.



Emmy Noether.

Chapter 4

Fields, A Wonderful Legacy

We have now unveiled the name of that medal, but the suspense was not just a ruse. When John Charles Fields proposed the creation of the medal, the story of which we will cover below, he insisted that it should be as international and impersonal as possible, and that it should not be associated with any person, country or institution, except for the international congresses. His good intentions were thwarted by a posthumous homage from his colleagues who named the medal after him.

John Charles Fields was the valiant and untiring organiser of the international congress celebrated in Toronto in 1924. Seven years later, in February 1931, the organising committee of the congress, having published the minutes, closed the bank accounts. There were \$2,700 (Canadian) left unspent, which Fields proposed should be set aside to create a prize consisting of two medals that would be awarded at the international congresses. One year of intense work later, Fields' idea had the approval of the world's main mathematical societies, he had contracted a sculptor to design the medal and sought support for the project from the prime minister of Canada (which he, in fact, did not receive; Canadian officialdom turned its back on the prize for many years).

He still needed the approval of the international congress, which was to be held in Zurich in September 1932. Fields was confident of doing so due to his prestige and the support he had built up. But four months before the congress he suffered a severe heart attack. According to witnesses, on his death bed he stammered instructions to his lawyer to donate practically his entire wealth, which was more than \$47,000 (Canadian), to his proposed prize fund, and he gave the task of presenting the proposal at the congress in Zurich to his friend John L. Synge. We know what followed: the congress approved the proposal, although it was not without resistance.

There have been scientific prizes since ancient times, as there is nothing closer to the intellectual and emotional heart of creation than a challenge. Institutionalised prizes have existed since at least the 18th century, created by scientific academies and

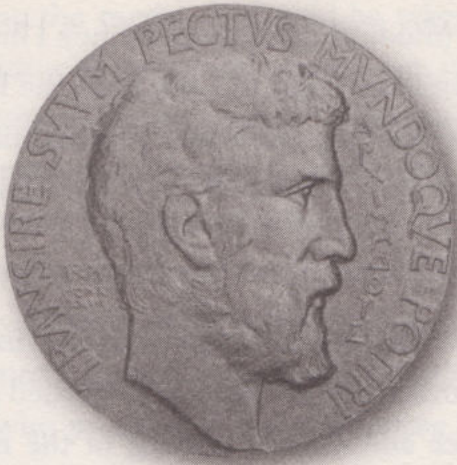
societies. But there has always been a certain scepticism in mathematics. The French mathematician Alain Connes, who received the Fields Medal at the congress held in Warsaw in 1982–1983, explained that:

“The usefulness of the prize is a delicate subject. In principle, the Fields medal spurs on research. But if it attracts a lot of publicity, it can have negative effects. Jean-Paul Sartre, when he received the Nobel Prize for literature, said that receiving a prize is like a kiss of death, as if it is not dealt with correctly it can destroy a person. Unfortunately, in some cases this is true.”

A well-known example of someone rejecting the prize is Alexander Grothendieck, who received the Fields Medal at the Moscow congress in 1966, although he did not collect it personally. Years later, Grothendieck received another prestigious prize and significant sum from the Swedish Academy of Sciences, which he rejected in a letter explaining that his salary was more than sufficient for his material needs and that he did not approve of the continuous and grave decline of the ethics of the scientific community.

A brief further note on John Charles Fields: he was born in 1863 in Ontario, Canada, in a family of Scottish and Irish origins. He studied mathematics at the University of Toronto and, because at the time it was not possible to obtain a doctorate in mathematics in Canada, he took his doctorate at Johns Hopkins University in the USA. In 1892 he left for Europe, where, thanks to his “simple living and abstemious habits,” he attended advanced courses given by the great mathematicians of the time at the universities of Paris, Berlin and Göttingen. He later returned to the University of Toronto. The experience in Europe, along with his friendship with Mittag-Leffler, gave him an interest in international cooperation in mathematics, which flourished with the organisation of the international congress in Toronto in 1924 – and in the creation of the medal that bears his name.

The Fields Medal was the first prize in mathematics that was open to the whole world, without any restrictions regarding area or nationality. It is awarded every four years at the international congress of mathematicians for “outstanding achievements” in mathematics. The winners are chosen by an international committee, named every four years, which remains secret until the last minute. It is the most prestigious international prize in mathematics.



John Charles Fields.

Top, front and back of the medal that bears his name.

THE NOBEL PRIZE FOR MATHEMATICS

There is no clear and convincing explanation for Alfred Nobel's exclusion of mathematics in establishing the prizes associated with his name. The Swedish mathematicians Lars Gårding and Lars Hörmander (the latter won a Fields Medal in 1962) have categorised the explanations we have. Firstly, there is the Franco-American version, according to which the mathematician Gösta Mittag-Leffler had a romance with Nobel's wife; and secondly, the Swedish version, which attributes it to the rivalry between Nobel and Mittag-Leffler. Gårding and Hörmander conclude that both versions are incorrect, as Nobel was never married and he barely had any contact with Mittag-Leffler.

The prize consists of a gold medal and a (modest) quantity of money. The medal has a bust of Archimedes on the front with an inscription in Latin from the Roman writer Manilius: "To pass beyond your understanding and make yourself master of the universe." The sculptor who created the medal (Canadian Robert Tait McKenzie) said of the portrait of Archimedes:

"I feel a certain amount of complacency in having at last given to the mathematical world a version of Archimedes that is not decrepit, bald-headed and myopic, but which has the fine presence and assured bearing of the man who defied the power of Rome."

The back of the medal reads, in Latin, "Mathematicians gathered from the entire world have awarded (this prize) for outstanding writings." Behind it there is a sphere inscribed in a cylinder, representing the drawing which, according to Cicero and Plutarch, Archimedes had engraved on his tombstone and which represents the method he used to calculate the volume of the sphere. The name of the winner is engraved around the edge of the medal. Curiously, the medal has a 'mathematical' error. The year in which the medal was cut, 1933, should appear in Roman numerals but, instead, "MCNXXXIII" is written because an M was swapped for an N.

In 1936, and at each of the following congresses until 1962, two medals were awarded. At the 1966 congress four were awarded in order to reflect with greater precision the development in mathematics every four years, thanks to the financial support of an anonymous donor. Since then between two and four medals have been awarded at each congress, depending on the judgement of the committee. (As we will see later, the 'austerity' of mathematicians has led to fewer than four medals being awarded on several occasions.)

The economic prize was, from the beginning, \$1,500 (which would possibly not even cover the cost of the journey to collect it). The 1974 congress was in Canada again, Vancouver this time, and the leftover money was used as the pot for the medals. Since then the amount has steadily increased to its current figure of \$15,000 Canadian dollars.

So far we have not touched on a strange fact that sets the Fields Medal apart from all other scientific prizes which are awarded worldwide. To win it you have to be younger than 40. The origin of this restriction lies in the memo written by Fields when he created the prize. Among the conditions, he explained that, "When

awarding the medals it should be taken into account that although it is done in recognition of the work carried out, it should also serve as inspiration for future achievements by the winners.” This was interpreted from the start as an allusion to young mathematicians, and in the end it resulted in the 40-year rule. Behind this requirement lies the conviction that truly difficult mathematical problems can only be solved by the power and creativity of a young mind.

The diversity of the mathematical results that have been deemed worthy of the Fields medal is enormous. In this sense, the words of German mathematician Heinz Hopf, who presided at the committee which awarded the Fields Medals in 1958, are fitting:

“The great variety within mathematics is due not only to the multiplicity of the branches of mathematics, but also to the diversity of the general tasks that face a mathematician in any branch. A task that is particularly fundamental is to solve old problems; and another, no less fundamental, is to open the way to new developments.”

ANDREW WILES AND THE FIELDS MEDAL

Fermat's last theorem is a simple conjecture proposed by the French mathematician Pierre Fermat in around 1637: The equation $x^n + y^n = z^n$ has no trivial positive integer solutions (except for zeroes and ones) when n is a natural number of greater than or equal to 3 (for $n=2$ there are solutions, the Pythagorean triples, 3, 4, 5, for example). Fermat wrote his conjecture in the margin of the book *Arithmetica* by the Greek mathematician Diophantus of Alexandria, adding that the lack of space meant he was not able to provide the proof. Since then mathematicians the world over have tried to solve the conjecture, only obtaining partial results.

At the international congress of 1994, the English mathematician Andrew Wiles gave a lecture in which he demonstrated a proof of the conjecture, but clarified that “one of the steps of the argument is not complete”, therefore the resolution was not certain. After the congress, a few weeks later, he achieved the complete proof, which he demonstrated at the following international congress, in 1998. At that time, Wiles was already 45 years old, so he could not receive a Fields Medal, despite the fact that the problem that he had resolved had been unsolved for more than 350 years. He was awarded a silver commemorative plaque instead.

MATHEMATICIANS WHO HAVE RECEIVED THE FIELDS MEDAL

Listed here are the year in which the medal was received, the name of the winner, their country of birth (even if it no longer exists) and their age on 1 January in the year they received the medal.

Year	Name of the winner	Country	Age
1936	Lars V. Ahlfors	Finland	29
	Jesse Douglas	USA	38
1950	Laurent Schwartz	France	34
	Atle Selberg	Norway	32
1954	Kunihiko Kodaira	Japan	38
	Jean-Pierre Serre	France	27
1958	Klaus F. Roth	Germany	32
	René Thom	France	34
1962	Lars Hörmander	Sweden	30
	John W. Milnor	USA	30
1966	Michael F. Atiyah	Great Britain	36
	Paul J. Cohen	USA	31
	Alexander Grothendieck	Germany	37
	Stephen Smale	USA	35
1970	Alan Baker	Great Britain	30
	Heisuke Hironaka	Japan	38
	Sergéi Novikov	USSR	31
	John G. Thompson	USA	37
1974	Enrico Bombieri	Italy	33
	David B. Mumford	Great Britain	36
1978	Pierre René Deligne	Belgium	33
	Charles L. Fefferman	USA	28
	Gregori A. Margulis	USSR	31
	Daniel G. Quillen	USA	37

1982– 1983	Alain Connes	France	34
	William P. Thurston	USA	35
	Shing-Tung Yau	China	32
1986	Simon K. Donaldson	Great Britain	28
	Gerd Faltings	Germany	31
	Michael H. Freedman	USA	34
1990	Vladimir Drinfeld	USSR	35
	Vaughan F. R. Jones	New Zealand	37
	Shigefumi Mori	Japan	38
	Edward Witten	USA	38
1994	Jean Bourgain	Belgium	39
	Pierre-Louis Lions	France	37
	Jean-Christophe Yoccoz	France	36
	Efim Zelmanov	USSR	38
1998	Richard E. Borcherds	South Africa	38
	W. Timothy Gowers	Great Britain	34
	Maxim Kontsevich	USSR	33
	Curtis T. McMullen	USA	39
2002	Laurent Lafforgue	France	35
	Vladimir Voevodsky	USSR	35
2006	Andrei Okounkov	USSR	36
	Grigori Perelman	USSR	39
	Terence Tao	Australia	30
	Wendelin Werner	Germany	37
2010	Elon Lindenstrauss	Israel	39
	Ngô Bào Châu	Vietnam	37
	Stanislav Smirnov	USSR	39
	Cédric Villani	France	36

The Nevanlinna Prize

The Nevanlinna Prize was established in 1982 in honour of the Finnish mathematician Rolf Nevanlinna. It is awarded, under conditions similar to that of the Fields Medal, for outstanding results in information sciences of mathematics (computing, complexity, programming, cryptography, computational algebra, etc.).

The Gauss Prize

Since the international congress held in Madrid in 2006 the Gauss Prize has been awarded when the international congresses are held. The prize was created “to highlight to the world the fundamental fact that mathematics is currently one of the driving forces behind modern technology”. It is awarded to scientists whose mathematical research has had an impact outside of mathematics, be it in technology, business or simply in people’s everyday lives. It is associated with the German mathematician Carl Friedrich Gauss, who throughout his life masterfully combined the abstract essence of pure mathematics with practical applications. As the applications of mathematics often require a long period of time, this prize has no age limit.

Chapter 5

Post-War Gains

The invasion of Poland in September 1939 by Hitler's Third Reich led to the outbreak of World War II and, therefore, the international congress planned for early September 1940 in the United States was cancelled. After almost six years of devastating war, in 1945 the world had to face a post-war period again, as it had done 27 years earlier, only this time the consequences of the war in terms of atrocities, deaths and destruction were beyond the limits of the imagination.

The lessons learned from the unfortunate repercussions the previous post-war period had on science helped to deal with it differently. Thus, the American Mathematical Society insisted from the start that "No international congress should be held if the meeting is not genuinely international, in the sense that any mathematician may attend regardless of their nationality or geographic origin."

The matter of the International Mathematical Union was also pending. At the Oslo congress in 1936, as no unanimous conclusion was reached on its future, the union was dissolved. It is worth noting that this is the only international scientific union created after the Great War that then disappeared, in this case abandoned by the mathematical community because of its sectarianism. After World War II, work started on the creation of a new union that was not to be a rehash of the previous one and was free from historical baggage. The US mathematician Marshall H. Stone consulted most of the world's mathematical societies. The most delicate subject was, of course, whether the German and Japanese mathematicians would participate. Despite the grave consequences of the war, including the Jewish Holocaust, it was generally agreed that this time there could be no exclusions.

All these intentions and actions led to the organisation of the international congress in August 1950 at Harvard University, in Cambridge, Massachusetts, and, simultaneously, a convention at Columbia University, in New York, for the constitution of a new International Mathematical Union (IMU).

The American way

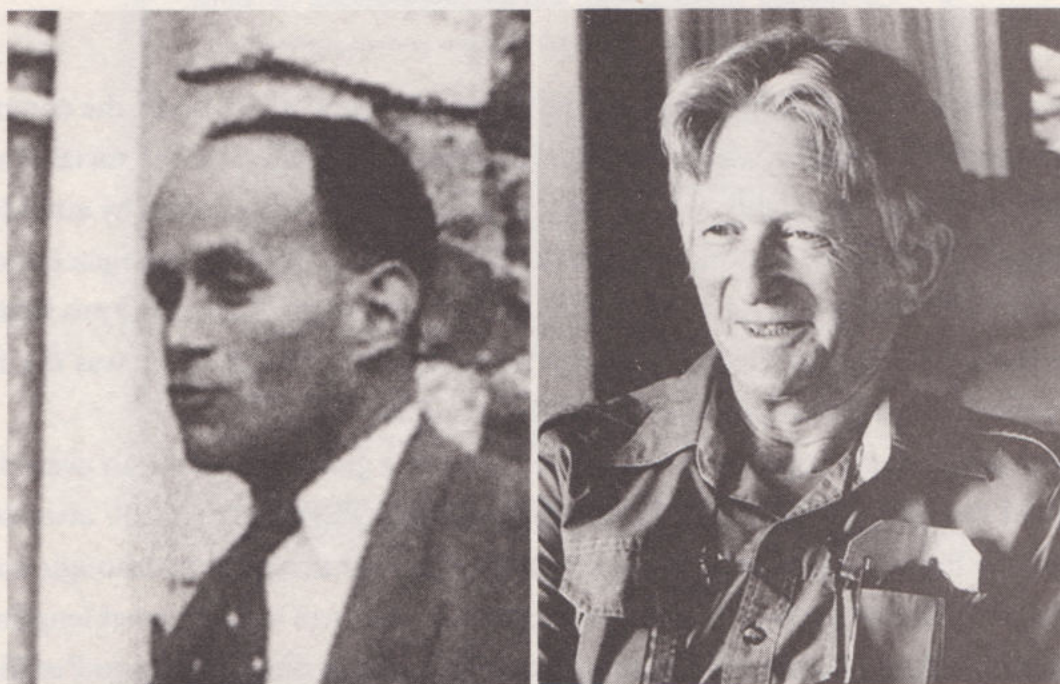
After 14 years without international congresses, the Harvard congress in 1950 was launched with great impetus and an intense scientific programme. There were 14 plenary conferences (most of them in English) in the first three days, with new names (Beurling, Hopf, Bochner, Gödel, Whitney, Hodge, Davenport, Schwartz) and innovative subjects, such as, for example, 'Distributions and Principal Applications'. In the scientific sections, similarly to the two previous congresses, there were 374 short announcements. Apart from this, each head of section had the opportunity to make an invitation to two or three relevant mathematicians to give special lectures on pertinent themes – there were 22 of these. Finally, following the model of the new congresses specific to certain fields – (there was one on topology in Moscow and another on probability in Zurich) – four special meetings were organised on fields that were going through periods of 'vigorous advances', namely algebra, applied mathematics, topology and analysis. Eight lectures were given on these subjects, followed by an open discussion. An excellent, but undoubtedly tiring programme.



Harvard University.

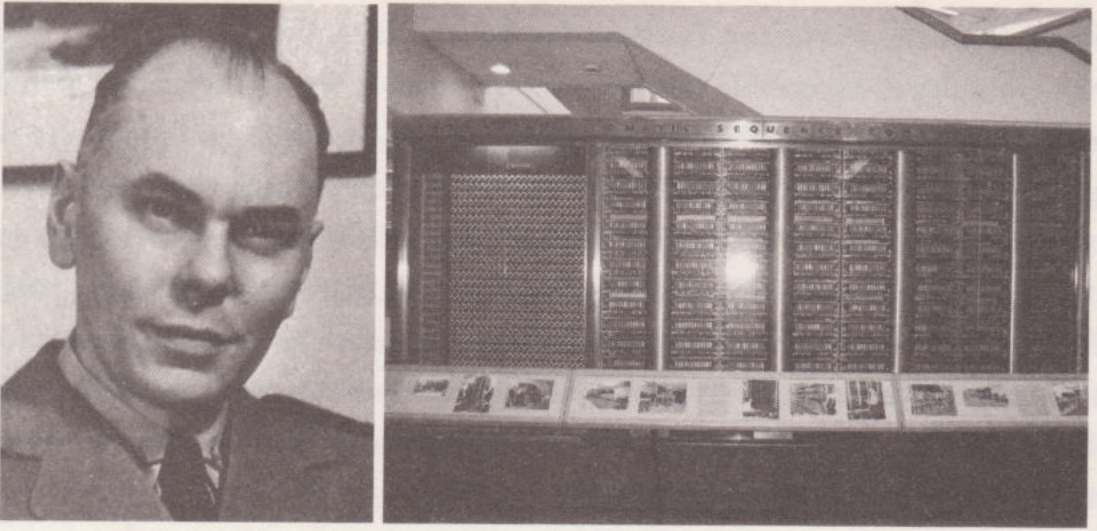
During the opening ceremony, the prize-giving ceremony for the Fields Medals took place for the second time. The winners were the French mathematician Laurent Schwartz, from the University of Nancy, and the Norwegian mathematician Atle

Selberg, from the Princeton Institute for Advanced Study. The president of the Fields committee, responsible for awarding the medals, was Danish mathematician Harald Bohr (brother of Niels Bohr, winner of the Nobel prize for physics). He explained the work of Schwartz and his “new ideas in their purity and generality” on the theory of distribution, reflecting the words of Felix Klein: “In our science new progress is often achieved by applying new methods to old problems”. Regarding Selberg’s work on the Zeta function of Riemann and an elementary proof of the prime number theorem, Bohr resorted to the words of Englishman G.H. Hardy: “No elementary proof of the prime number theorem is known, and one may ask whether it is reasonable to expect one.”



Laurent Schwartz (left) and Atle Selberg, the two winners of the Fields medals awarded at the 1950 congress.

Showing a fine balance between past and future, the congress, on the one hand, named three honorary presidents: Guido Castelnuovo of the Accademia dei Lincei in Rome; Jacques Hadamard of the Collège de France in Paris; and Charles de la Vallée-Poussin of the Université Catholique de Louvain in Belgium. The combined age of the three was 254. On the other hand, it organised a conference by Commander Howard Aiken, designer of the Harvard Mark series of electro-mechanical computers, built in collaboration with IBM.



Commander Howard Aiken and the Harvard Mark I computer.

In a certain sense, to which we referred at the beginning of the book, the congress inaugurated a new model, which is evident in something as lateral as the social events. On the one hand, the quality and variety of the music on offer, chosen by attendees, was excellent. There was a concert from the Busch String Quartet, an organ concert at Boston University, a concert of ballads by the folk singer Richard Dyer-Bennet and a recital by soprano Helen Traubel. And on the other hand, there was the first beer party ever organised at an international congress.

Everything came together to make the congress a genuine success, an event that breathed new life into the international congresses. There were 1,700 attendees, double the highest figure attained at any of the pre-war congresses, although many came from different parts of North America. The organisers went to great lengths to help boost the attendance of non-Americans, including a direct appeal to President Truman for visas for Jacques Hadamard (honorary president of the congress, who at 84 years of age continued to be a communist sympathiser) and Laurent Schwartz (a Fields Medallist and self-confessed Trotskyist).

Attendance by mathematicians from the Soviet Union is a case apart. The president of the Academy of the USSR sent the following cable to the congress:

“The USSR Academy of Sciences is grateful for the kind invitation to Soviet scientists to attend the International Congress of Mathematicians to be held in Cambridge. The Soviet mathematicians are very busy with their regular work and cannot attend the congress. We wish you success with the activities at the congress.”

SECRETARY'S REPORT

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There were in attendance at the Congress 2,302 persons. These people were distributed as follows:

United States and Canadian.....	1,410
United States and Canadian, associate members.....	540
Members outside United States and Canada.....	290
Associate members outside the United States and Canada.....	78
Total members in attendance.....	2,316

In addition there were a number of mathematicians who became members of the Congress but did not attend the gathering at Cambridge. Some of these persons joined fully expecting to be present, but were prevented from attending either by illness or other unavoidable circumstances, while others assumed membership to support an enterprise which they considered most worthy. The statistics in this connection are as follows:

United States and Canadian members unable to attend.....	160
United States and Canadian associate members unable to attend.....	35
Members outside United States and Canada unable to attend.....	57
Associate members outside United States and Canada unable to attend.....	3
Total members unable to attend.....	255

Total Congress membership..... 2,571

The following countries outside the United States and Canada were represented:

Argentina, Australia, Austria, Belgium, Brazil, Burma, Chile, China, Colombia, Cuba, Denmark, Egypt, England, Finland, France, Germany, Greece, India, Iran, Ireland, Israel, Italy, Japan, Mexico, Netherlands, Nigeria, Norway, Panama, Peru, Philippines, Scotland, South Africa, Spain, Sweden, Switzerland, Turkey, Uruguay, Venezuela, and Yugoslavia.

Every state in the United States and the District of Columbia were represented, with the exception of South Dakota.

THE SCIENTIFIC PROGRAM

In addition to the contributed ten-minute papers and the stated addresses, which were delivered on invitation of the Organizing Committee, the scientific program of the Congress included four conferences on the following topics: Algebra, Analysis, Applied Mathematics, and Topology. The chairman of each section for contributed papers was given the privilege of inviting not more than three persons to deliver thirty minute addresses in his section.

Participants at the 1950 congress in Cambridge, Massachusetts.

The congress clearly reflected the new post-war economic and social model. Although most of the financial support came from the Carnegie Corporation, the Rockefeller Foundation, the American mathematical societies and the local universities (Harvard, Boston, MIT, Princeton, Yale), there were significant contributions from all kinds of companies (Ford, General Motors, Shell, Standard Oil, General Electric, Bell, Eastman Kodak, United Fruit, etc.).

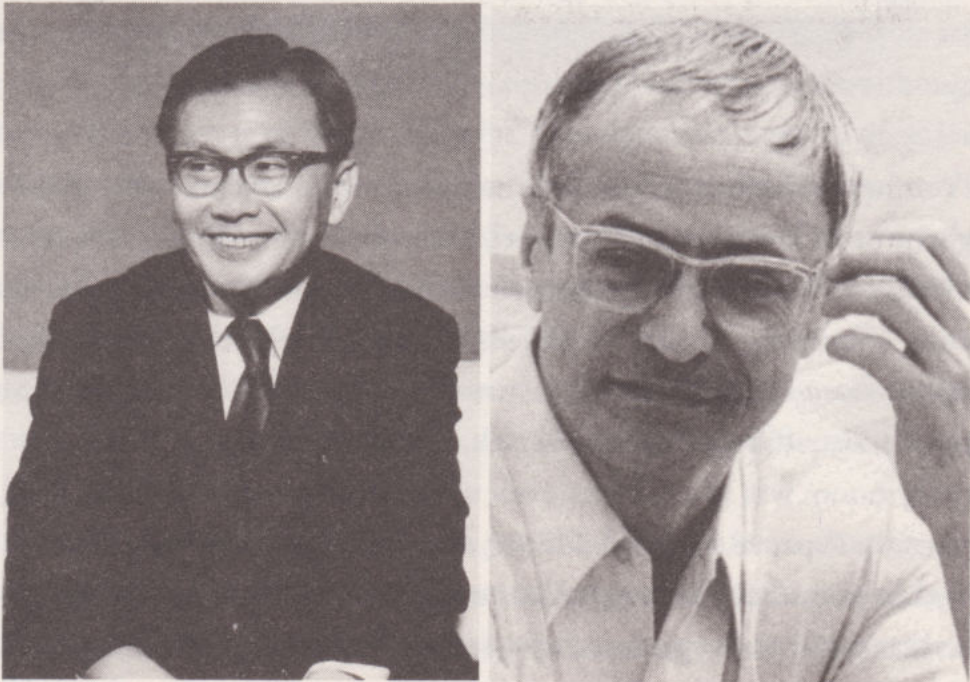
The baby boom was also evident, even in the advertising for the congress, which informed of the "special measures for the care of children", something that can be appreciated in photos of the event. And an authentic advert for the American way of life was created by the organisers, who recommended that "Mathematicians who can, help out by making their cars available to the social events committee for trips outside of Cambridge."

Three great congresses

The following international congresses were held in Amsterdam in 1954, Edinburgh in 1958 and Stockholm in 1962. Following the success of the first post-war congress at Harvard, the basic model for the international congresses was kept the same. A classic but restrained style, typical of post-war Europe, can be appreciated in them.

The Amsterdam congress was inaugurated in a hall full of national flags in the Concertgebouw, the city's splendid concert hall. A speech by Jan A. Schouten, the president of the Dutch mathematical society, the Wiskundig Genootschap, in Dutch, English, French, German, Italian, Swedish and Russian, was followed by a musical interlude consisting of piano music by Chopin and, then, the much-anticipated awarding of the Fields Medals took place.

The president of the Fields committee, German mathematician Hermann Weyl, unveiled the names of the winners: the Japanese mathematician Kunihiko Kodaira, from Princeton University, and the Frenchman Jean-Pierre Serre, from the Collège de France. As Weyl explained, Kodaira had achieved "outstanding results in the theory of harmonic integrals and numerous applications... to the study of algebraic varieties". As for Serre, his contributions to the theory of the homotopy of spheres and "the abundance of his surprisingly numerical results" set him apart.



Kunihiko Kodaira (left) and Jean-Pierre Serre.

The mathematical content of the congress was divided into 20 one-hour plenary conferences, 42 half-hour conferences associated with the various sections and 496 short announcements within those sections. The two opening and closing plenary lectures at the congress were given by two renowned mathematicians. The American John von Neumann, spoke on 'Unsolved Problems in Mathematics', and the Soviet Andréi Nikoleyevich Kolmogorov dealt with the 'Théorie générale des systèmes dynamiques et mécanique classique'.



John von Neumann and Andréi Nikoleyevich Kolmogorov.

In relation to the congress, three specific symposia were also organised on 'Stochastic processes', 'Algebraic geometry' and 'Mathematical interpretation of formal systems'. The organisation necessary to accommodate so much mathematics required an interesting system in each room. The speaker was informed by a traffic light when there were only two minutes left (yellow) and when time was up and they had to finish (red).

For the first time at an international congress there was a demonstration of the uses of several 'electronic' devices:

- IBM's 604 and 626 calculators.
- The ARRA electronic computer, built at the Amsterdam Mathematics Centre.

— ‘Miracle’, the electronic computer built by Ferranti and belonging to Royal Shell Research Laboratories (B.P.M.) in Amsterdam.

The counterpoint of these demonstrations was provided by the prodigious human calculator Wim Klein, known as Pascal, who demonstrated his abilities during an evening at the Natura Artis Magistra Zoo in Amsterdam where his calculations were simultaneously verified by an electronic calculator.

	A	B	C
Algeria	—	3	—
Argentina	10	10	10
Australia	3	1	—
Austria	19	15	13
Belgium	27	29	26
Brazil	3	3	3
Bulgaria	1	1	1
Canada	17	18	11
China	4	1	1
Columbia	—	1	—
Cuba	1	—	—
Czecho-Slovakia	4	4	4
Denmark	14	18	14
Egypt	3	2	2
Finland	9	9	9
France	138	168	128
Germany	207	212	197
Gold-Coast	—	2	—
Great-Britain	261	261	237
Greece	11	8	8
Hungary	3	2	2
Iceland	1	1	1
India	18	12	11
Indonesia	2	3	2
Iran	3	2	2
Ireland	11	16	11
Israel	10	10	8
Italy	89	91	87
Japan	10	3	3
Latvia	1	—	—
Luxemburg	2	2	2
Mexico	3	3	2
The Netherlands	212	223	210
Nigeria	2	1	1
Norway	21	20	20
Pakistan	2	3	2
Peru	2	2	2
Philippines	2	2	2
Poland	7	6	6

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	A	B	C
Portugal	5	4	4
Roumania	5	4	4
Russia	5	5	5
Siam	1	—	—
Soudan	—	1	—
South-Africa	10	5	4
Spain	8	8	8
Sweden	39	41	38
Switzerland	41	52	38
Tunisia	—	3	—
Turkey	12	12	10
United States of America	228	242	198
Uruguay	1	2	1
Venezuela	1	1	1
Vietnam	1	—	—
West-Africa	—	2	—
Yugoslavia	15	13	13
Stateless	12		
No statement	36		

1553	1553	1362
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For organizational purposes all participants were requested to state whether they were able to understand English, French or German.

The following list gives the numbers of those regular members speaking or understanding the languages mentioned in column 1. The last column refers to associate members only.

Language(s)	number of men	number of women	total	ass. members
French	64	2	66	67
German	34	3	37	26
English	189	16	205	133
French, German	36	2	38	5
German, English	148	9	157	67
French, English	193	29	222	84
French, German, English	339	32	371	71
No statement	433	24	457	115

The following list may illustrate the distribution of the regular members over the main professions:

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Participants by country in 1954.

The high point of the cultural aspect of the congress (besides the wonderful concerts at the Concertgebouw) was the first great exhibition by the Dutch artist Maurits Cornelis Escher. It was organised by the international congress and the Stedelijk Museum in Amsterdam because Escher’s work: “Shows many mathematical tendencies and is connected with mathematical thinking in surprising ways.” The

A CLOTH OF COMPLEX PRIMES

For the 1954 congress, the company Van Dissel & Zn wove some linen cloth with a design by the Dutch mathematician Balthasar van der Pol showing the Gaussian primes on the complex plane. They were sold during the congress and can still be purchased at the Van Dissel & Zn shop in Eindhoven.

exposition was a success, both in terms of the public, and the relationships that the artist forged with significant scientists, such as the Canadian geometrician H.S.M. Coxeter and the physicist Roger Penrose.

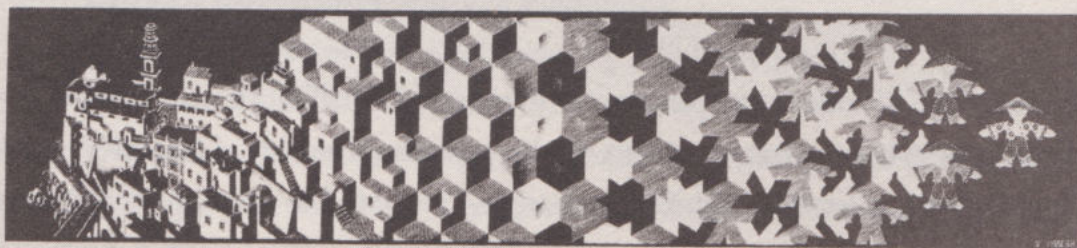
In the exhibition's catalogue, Dutchman Nicolaas Govert de Bruijn attempted to explain the attraction of Escher's work to a mathematician:

"The mathematicians will probably not only be interested in the geometric motifs, the same spirit of fair play that is ever-present in mathematics and which for many mathematicians it is the charm of the field itself, will be a more significant element. Members of the congress will greatly enjoy themselves when they recognise their own ideas, interpreted in a different medium to that which they are accustomed."

MAURITS CORNELIS ESCHER (1898–1972)

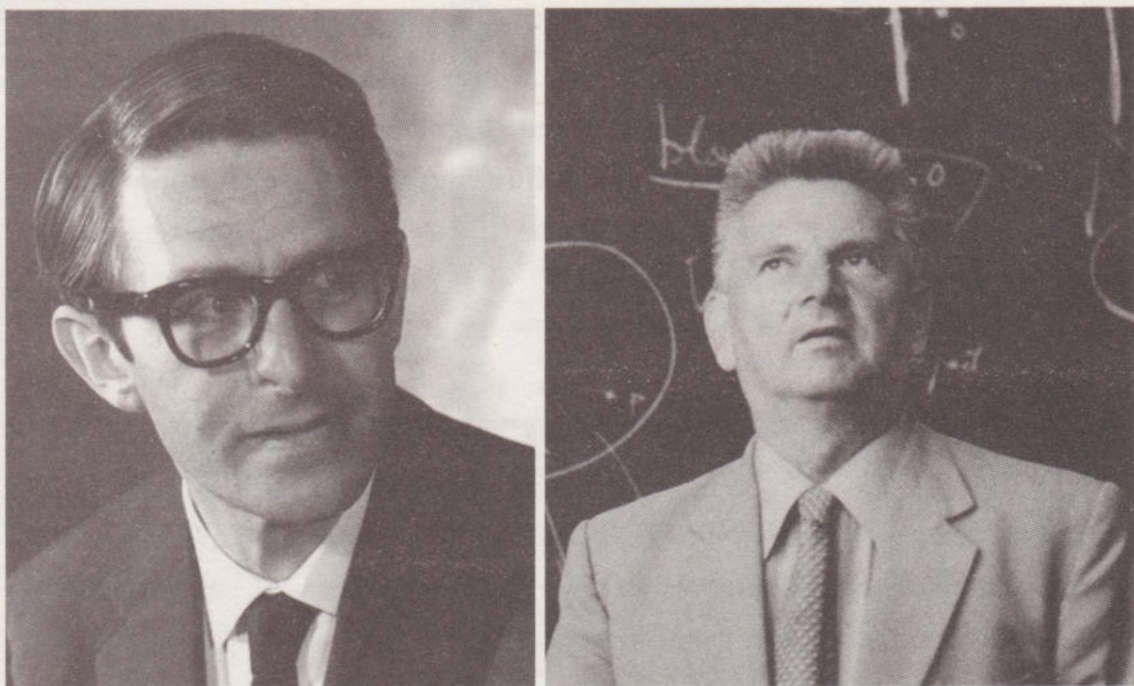
M.C. Escher visited the Alhambra Palace in Granada and the Mosque of Cordoba in the 1920s and 30s. He was fascinated by the mosaics created by the medieval Islamic craftsmen. He later maintained a correspondence with the Hungarian mathematician George Pólya with reference to groups of symmetry on the plane, which can appear in mosaics.

Below: Metamorphosis I (1937).



The 1958 congress was inaugurated in the sturdy MacEwan Hall at Edinburgh University with a memorial service to English mathematician Sir Edmund Whittaker and a message from the Duke of Edinburgh, patron of the congress, from Buckingham Palace.

Then the Fields Medals were awarded, this time to the German mathematician Klaus Friedrich Roth, from London University, and to the Frenchman René Thom, from Strasbourg University. The complexity of the subjects involved led to a change in the protocol of awarding the medals, such that those who summarised the work of Roth (resolving “the main problem of the approximation of algebraic numbers by rational numbers”) and Thom (creator of the cobordism theorem, which “has strongly influenced topology and, through topology, other branches of mathematics”) were necessarily specialists in those areas.



Klaus Roth (left) and René Thom, the two mathematicians awarded the Fields Medal in 1958.

There were 19 one-hour plenary conferences, 37 half-hour conferences in the sections and 604 short presentations in the sections. The list of scientific sections at the congress underwent significant changes compared to previous congresses, in order to reflect the new areas being researched.

- I. Logic and fundamentals.
- II (a). Algebra.
- II (b). Theory of numbers.
- III (a). Classic analysis.
- III (b). Functional analysis.
- IV. Topology.
- V (a). Algebraic geometry.
- V (b). Differential geometry.
- VI. Probability and statistics.
- VII (a). Applied mathematics.
- VII (b). Mathematical physics.
- VII (c). Numerical analysis.
- VIII. History and education.

Among the papers in section VIII on history and education was one titled 'Teaching Mathematics on Television'.

A significant change at the Edinburgh congress was the participation of the USSR. The Soviet mathematicians had been absent from the international congresses since 1932, with just one attendee in Amsterdam in 1954. But the death of Stalin in 1953, and the subsequent process of destalinisation precipitated a change. In 1957, the Soviet Union, and other communist countries also, joined the International Mathematical Union, which had been founded in 1951 with ten countries (Germany, Austria, Denmark, France, the UK, Greece, Holland, Italy, Japan and Norway), to which many more were subsequently added. This meant that the Soviet delegation in Edinburgh was somewhat more of a reflection of the country's mathematical prowess, 35 mathematicians attended. Three of them gave plenary conferences:

- 'Modern Development of Surface Theory', by A.D. Alexandrov.
- 'On Some Mathematical Problems of Quantum Field Theory', by N.N. Bogolyubov and V.S. Vladimirov.
- 'Optimal Processes of Regulation', by L.S. Pontryagin.

The local feel to the congress was provided in the evening, with Scottish dancing, a steamboat trip down the River Clyde, around the Isle of Bute and on to Gourock. The Soviet presence at the congress led to a few jokes, as Paul Halmos, the Hungarian

mathematician based in the US, remembered: "I don't think *everyone* on the excursion up the Clyde was Soviet."

In 1912 the Swedish mathematician Gösta Mittag-Leffler proposed that the International Congress of Mathematicians be held in Stockholm in 1916, an idea that had been accepted, but the war prevented it from taking place. In 1962, 46 years later, Gösta Mittag-Leffler's dream came true.

Holding the congress in Sweden was an *in extremis* solution to the problem that arose at the end of the Edinburgh congress as there were several competing proposals for the 1962 host, among which was Hebreia University in Jerusalem. From the point of view of mathematical solvency, Israel was the country best qualified to organise an international congress, but the political situation generated by the Palestine-Israel conflict made it very difficult.

The inauguration was, as in the previous congresses of this period, grandiose. It was held in the Konserthuset in Stockholm in the presence of King Gustaf VI Adolf of Sweden and a full orchestra. It was the king who personally awarded the Fields medals to the winners, who were the Swedish mathematician Lars Hörmander, from Stockholm University, for his "outstanding work on partial differential equations", and US mathematician John Milnor, from Princeton University, for his contributions to differential topology, "the vitality of which is largely owed to Milnor's stupendous results".



The 1962 Fields Medals were awarded to Lars Hörmander (left) and the American John Milnor.

The scientific programme maintained the increase in the number of contributions: 16 plenary conferences, 57 lectures in the sections and 745 short presentations, which continued to create difficult management problems.

A glance at the titles of the plenary conferences allows us to see how things had changed from the first congresses, where the titles were clearly descriptive and simple, and even poetic on occasions. Now they were much more technical:

- ‘Teichmüller Spaces’, by L.V. Ahlfors.
- ‘Arithmetic Properties of Linear Algebraic Groups’, by A. Borel.
- ‘Logic, Arithmetic and Automata’, by A. Church.
- ‘Markov Processes and Problems in Analysis’, by E.B. Dynkin.
- ‘Homotopy and Cohomology Theory’, by B. Eckmann.
- ‘Automorphic Functions and the Theory of Representations’, by I.M. Gelfand.
- ‘Die Bedeutung des Levischen Problems für die analytische und algebraische Geometrie’, by H. Grauert.
- ‘Problems of Stability and Error Propagation in the Numerical Integration of Ordinary Differential Equations’, by P. Henrici.
- ‘Transformés de Fourier des fonctions sommables’, by J.-P. Kahane.
- ‘Topological Manifolds and Smooth Manifolds’, by J. Milnor.
- ‘Geometrical Topology’, by M.H.A. Newmann.
- ‘Some Aspects of Linear and Nonlinear Partial Differential Equations’, by L. Nirenberg.
- ‘Algebraic Number Fields’, by I.R. Safarevich.
- ‘Discontinuous Groups and Harmonic Analysis’, by A. Selberg.
- ‘Géométrie algébrique’, by J.-P. Serre.
- ‘Groupes simples et géométries associées’, by J. Tits.

One of the interesting complementary activities at the congress was Sergei Lvovich Sobolev’s lecture on the use of electronic computers at Novosibirsk University in Siberia in the struggle to decipher Mayan writing.

2,107 mathematicians attended the Stockholm congress. As its organisers said, it was “Without doubt one the greatest groups of people ever brought together in the history of the world to talk about mathematical research.” They came from 57 countries and, including wives and husbands, numbered 3,091 people. This generated debate in the international mathematical community on whether these congresses

could continue to be organised, given the rate of growth in the attendances and the enormous organisational difficulties associated with them.

WORLD DIRECTORY OF MATHEMATICIANS

The World Directory of Mathematicians, an old idea that was first expressed at the 1897 congress in Zurich, was presented at the Stockholm congress. Compiling a list with the name, address, position and area of interest of all the active mathematicians in the world was no easy task. It was carried out by the International Mathematical Union with the help of the Tata Institute of Fundamental Research in Bombay. It was updated every four years until 2002.

The Scottish mathematician William Vallance Douglas Hodge had made his contribution to this debate when, at the closing ceremony of the Edinburgh congress in 1958, of which he was president, he said:

“Through the choice of the invited speakers and through the large number of papers of other members, the congress has presented a picture of mathematics today with all its trends. But the international congresses have another purpose, which I believe is just as important, that of promoting the fellowship between mathematicians of all countries. This fellowship has its roots in our common love for our science, to whose growth we all try to contribute. It is the responsibility of each generation to take care that this fellowship is maintained and strengthened, and extended to the new generation.”

Chapter 6

The Chill of the Cold War

The USSR's initial participation at Edinburgh in 1958 opened the door to the incorporation of the vast and powerful Soviet mathematical community into the international congresses. Holding the international congress in Moscow in 1966 was another milestone en route to the modern congresses organised today. The new world order created by World War II was becoming patently evident in them, as the congresses went from being select reunions for privileged scholars to mass meetings of scientists. It was also the start of the period in which the long shadow of the Cold War was felt in international scientific cooperation.

The great communist congress

Yuri Gagarin and Valentina Tereshkova's journeys into space were timely proof of the power and bold future of Soviet science and communism. Holding the international congress in Moscow in 1966 provided the perfect showcase for the Soviet system. And the world was reminded of it during the press conference that was held on every one of the eleven days of the congress – something that was completely new.

The congress smashed all the attendance records: there were 5,600 pre-inscriptions, of whom 4,280 materialised as actual attendees and 1,470 of whom were Soviets. It is not easy to recreate the emotion that a scientific community must have felt after 30 years in isolation, imagining contact with thousands of Western mathematicians. Soviet mathematicians who were young at the time still remember the excitement today.

Likewise, visiting 'the other side of the Iron Curtain' and seeing the large Soviet mathematical schools attracted many Western mathematicians. In terms of attendance, the countries that stood out were the USA with 725, 398 Germans (including Germans from the Federal Republic, the Democratic Republic and West Berlin), 286 Britons and 280 from France.

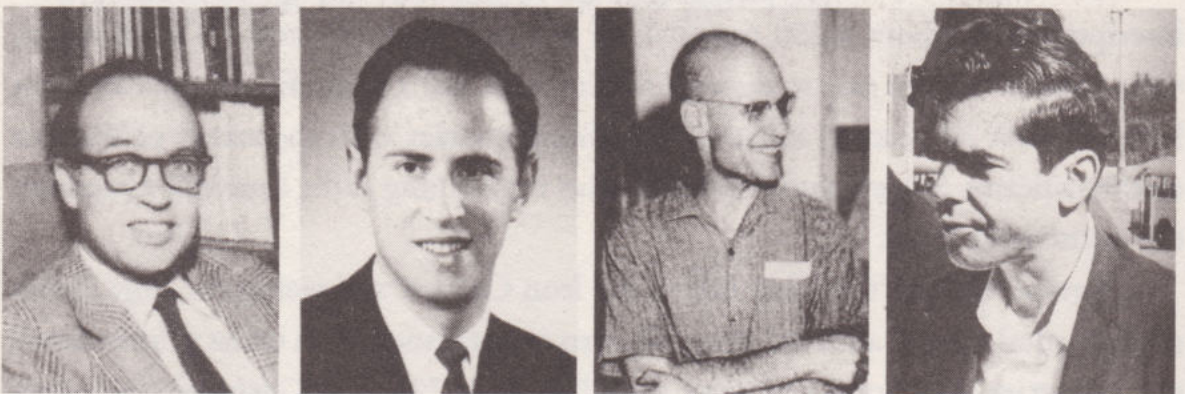
The opening ceremony was held in style at the Palace of Congresses, which had recently been built inside the Kremlin. Some of the top authorities of the Soviet scientific establishment were present, such as the academic Mstislav Vsevolodovich

Keldysh, head of the Soviet cosmonaut programme and president of the all-powerful USSR Academy of Sciences; the academic Ivan Georgievich Petrovsky, mathematician and rector of the Moscow State University, as well as other academics of great renown, such as the theory of numbers expert Ivan Matveyevich Vinogradov and the physicist-mathematician Mikhail Alekseevich Lavrentiev.

PARTICIPANT NUMBER 4,397

The same question could be heard repeated throughout the congress, regarding the participant registered under number 4,397: "Has Nicolas Bourbaki arrived in Moscow yet?" The mysterious French mathematician had signed up to the congress and could not be found anywhere.

The Fields Medals, of which there were four for the first time, were awarded at a packed ceremony. The winners were the English mathematician Michael F. Atiyah, from Oxford University, for "the K theory, the index formula and the Lefschetz formula"; the American mathematician Paul J. Cohen, from Stanford University, for the solution to Cantor's continuum problem, Hilbert's first problem; the German mathematician Alexander Grothendieck, from the University of Paris, for his renovation of algebraic geometry "eliminating parasitic restrictions", and US mathematician Stephen Smale, from the University of California, in Berkeley, for his work on the Poincaré conjecture.



The 1966 Fields Medallists in Moscow. From left to right, Michael Atiyah, Paul Cohen, Alexander Grothendieck and Stephen Smale.

Of the four medals only two could be handed over by the academic Keldysh, as Smale arrived to the ceremony late and was not in time to collect it, and Grothend-

ieck did not attend the congress, apparently in protest at the Soviet Union's treatment of dissidents (his father was an anarchist militant who suffered persecution from the Bolsheviks); his medal was received by Léon Motchane, the founder and director of the Institute of High Scientific Studies in Paris, IHES.

The congress was held in the impressive Moscow State University building, also known as the Lomonosov University and by its initials MGU. It is a lavish 40-storey skyscraper, built in the 1950s on the Lenin Hills in Moscow. There are six other similar buildings in the capital, and they are popularly known as the 'Seven Sisters'. The building houses the classrooms, administrative services, professors' living quarters and students' residences. In the campus stand the statues of two renowned Russian mathematicians: Nikolái Ivanovich Lobachevsky and Pafnuty Lvovich Tchebyshev.



Lomonosov University in Moscow, MGU.

Seventeen plenary conferences were planned (of them, five were addressed by Soviet mathematicians and another five by American mathematicians, a demonstration of the geostrategic balance) and 64 sectional conferences. The exact number of short communications is not known, as the minutes from this congress lack certain pieces of information, such as the list of papers in the sections and the list of participants. It is a shame as this information always provides interesting insights into a congress. We do know that 2,100 proposals were received for short talks. In fact, up to 40

classrooms spread around the building operated simultaneously with lectures and seminars. These numbers show how all the previous records were broken at this congress.

Among the plenary sessions was one given by Swedish mathematician Lennart Carleson, in which he resolved a problem on the convergence of the Fourier series. This problem dated back to 1913, when it was proposed by Russian mathematician Nikolai Nikolaevich Lusin. The transcendence of the result would have won Carleson a Fields Medal, but his age meant he was not a candidate. Lennart Carleson received the Abel Prize in 2006.

CONVERGENCE AND SUMMABILITY OF FOURIER SERIES

LENNART CARLESON

Let me first state quite explicitly that I do not intend to give in this lecture any survey of the very large field covered by the title. There is also no need for this since the Congress was presented such a survey quite recently. I rather want to present my personal interests which are concentrated on the almost everywhere behaviour of the partial sums. Also the subject of summability will only be touched upon.

1. Background

For a very long time, the outstanding result in the area of almost everywhere convergence has been the following result of Kolmogorov-Seliverstov-Plessner: if for $\lambda_n = \log n$

$$(1.1) \quad \sum_1^\infty (a_n^2 + b_n^2) \lambda_n < \infty,$$

then

$$(1.2) \quad s_n(x) = \frac{a_0}{2} + \sum_1^n (a_v \cos vx + b_v \sin vx)$$

converges a.e. The outstanding question was whether $\log n$ is a relevant sequence or not.

It has been known that conditions of the type (1.1) are related to capacities with respect to a kernel

$$(1.3) \quad K(x) \sim \sum \frac{\cos nx}{\lambda_n}$$

(Beurling [1]: $\lambda_n = n$, $K(x) \sim \log \frac{1}{|x|}$; Salem-Zygmund [4]: $\lambda_n = n^\alpha$, $K(x) \sim |x|^{\alpha-1}$). However, what they really prove is that the capacity of the divergence set vanishes for

$$K^*(x) = \frac{1}{|x|} \int_0^{|x|} K(t) dt$$

(see Temko [5]). When $\lambda_n = (\log n)^\beta$,

$$K(x) \sim |x|^{-1} \left(\log \frac{1}{|x|} \right)^{-1-\beta}$$

6*

THE ABEL PRIZE

The Abel Prize was created by the Norwegian Academy of Sciences in 2002 in memory of nineteenth century Norwegian mathematician Niels Henrik Abel. The prize was at the point of being created a century earlier, in 1902, for the centenary of Abel's birth. The driving force behind it was the Norwegian mathematician Sophus Lie, and it was intended as a response to the lack of a Nobel Prize for mathematics. Lie's plans had to wait one hundred years. Today it is a prestigious prize and its awarding and monetary prize are very similar to that of the Nobel Prize. The winners and details of the prize can be found at <http://www.abelprisen.no/en/>.

The congress was held in an informal, relaxed and friendly environment, as shown by the images that we have of groups sat on the stairs in animated conversation. Even today stories still circulate about the "mixture of mathematics, vodka, caviar and hangovers" which were common during the congress. The camaraderie was made evident by the football match in which the USSR took on the rest of the world – and won 5–2.

During the congress a bizarre event occurred that has come to be known as "the Smale incident". A professor at Berkeley, Stephen Smale was notable for his activism against the war in Vietnam, to the point where the US Un-American Activities Committee subpoenaed him. This led to a petition in support of him – and other US academics opposed to the war – being signed at the Moscow congress. Making the most of the situation, a North Vietnamese journalist requested a personal interview from Smale who, in order to guarantee publicity for his declarations, converted it into a press conference.

The press conference took place on the last day of the congress, on 26 August 1966, on the steps of Moscow University. There, before the gathered public and the international press, Smale launched a bitter attack on the US military intervention in Vietnam, showing his sympathy for the Vietnamese victims. But, as he subsequently explained, because he was in Moscow he felt the need to balance this declaration and made an emotional reference to the Hungarians who had died at the hands of Russian troops in 1956 when fighting for the freedom of their country.

As if appearing from a spy novel, two broad-shouldered Russian civil servants appeared and forced Smale into a car, which quickly sped off. The scene was immortalised by a press photograph and was front page news in the *New York Times* the next day.

Smale disappeared for the whole day, and returned when the closing ceremony had already finished. What happened during those hours forms part of the incomprehensible mysteries of Soviet bureaucracy. Smale was taken through the museums of Moscow and invited to the headquarters of the Soviet press agency. Although he later confessed to have been somewhat frightened, he was treated like a dignitary at all times.

HOLIDAY CAMP

Since 1950, the General Meeting of the International Mathematical Union has been held before the international congresses. All the member countries are represented and its organisation is the responsibility of the country organising the congress. However, there was a conflict in 1966 when the Soviet authorities proposed that the meeting be held in a holiday camp run by the Academy of Sciences. Memories from the gulags led to the proposal being rejected by the international community.

Bourbaki in action

The Great War had a profound impact on France and, particularly, on its youth, something that was also felt in the academic world, where the replacement of the older generation by a younger one was not without its impasses. In the 1930s, a group of French mathematicians from the École Normale Supérieure in Paris began to meet with the initial purpose of drawing up a basic text of mathematical analysis that renewed the fantastic, but now outdated, classic French texts. Their chief criticism was the style of Henri Poincaré and his emphasis on intuition.

The initial group was formed by the mathematicians Henri Cartan, Claude Chevalley, Jean Dieudonné, Charles Ehresmann, Szelem Mandelbrojt and André Weil, among others. New members joined (and also left) later, such as Laurent Schwartz, Jean-Pierre Serre, Alexander Grothendieck, Samuel Eilenberg, Serge Lang and Roger Godement. The group's first meetings were called in 1934. They agreed to use the name Nicolas Bourbaki, and for quite some time they behaved in a somewhat secretive manner. Jean Dieudonné came to be considered the group's spokesperson.



*Jean Dieudonné (left) and Henri Cartan, two of the founders of the Bourbaki group
(photo: Konrad Jacobs, Erlangen).*

What started as a specific project for a book became a programme to present all of mathematics in a structured format. Just as Euclid is supposed to have done in his time (if he existed) compiling and organising a logically ordered and rigorous presentation of the mathematical knowledge of his era, this group planned to write 'Elements of mathematics' which took into account the state of current mathematical knowledge. The main books in the project were:

- I. Théorie des ensembles.
- II. Algèbre.
- III. Topologie générale.
- IV. Fonctions d'une variable réelle.
- V. Espaces vectoriels topologiques.
- VI. Intégration.

These were followed by other books, which have had a lesser impact.

The Bourbaki group undoubtedly had a very healthy effect on 20th-century mathematics by extending its rigour to the darkest corners of theories and by demonstrating the structure of mathematical theories. But it had its limitations and many critics. Not all of the areas fit the Bourbaki style of presentation and organisation, and not all of the mathematics in a field fit that model. In any case, it proved to be a strong catalyst.

ON THE LONGEVITY OF MATHEMATICIANS

The Nice congress named the French mathematician Paul Montel, who was 94 years old at the time, as honorary president.

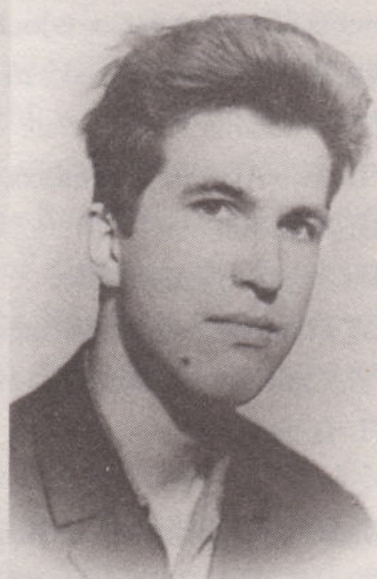
The 1970 international congress was held in Nice. As well as the traditional elements from the previous congresses, there were two new items in this one that set it apart from the rest.

The first of the innovations related to the conferences. There were 16 one-hour plenary conferences, two every morning. The rest of the morning was free. Afternoons were filled with 242 50-minute expository lectures from specialists in each field, organised in three parallel sessions of three hours each. The rest of the afternoon was available to the participants for discussions, for which a multitude of classrooms were prepared. There were no short presentations associated with the sections.

ONE OF HILBERT'S PROBLEMS RESOLVED

At the age of 23, a Soviet mathematician from Leningrad called Yuri V. Matijasevich, presented the (negative) solution to the tenth problem of those proposed by Hilbert in Paris in 1900. The problem was: "Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients, devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers."

Yuri Matijasevich in 1969, when he resolved Hilbert's tenth problem, just one year after the Nice congress.



This unusual organisation of the lectures was the result from a vision of mathematics that becomes clear when looking at the scientific sections of the congress.

Mathematics had been divided into seven fields, and these, in turn, were divided into a total of 34 sub-fields:

A. Logique Mathématique.

B. Algèbre.

- B1. Algèbre générale.
- B2. Catégories; algèbre homologique.
- B3. Groupes Finis.
- B4. Corps locaux et globaux; analyse p-adique.
- B5. Géométrie algébrique.
- B6. Théorie des nombres, élémentaire et analytique.

C. Géométrie et Topologie.

- C1. Topologie générale et algébrique.
- C2. Topologie des variétés.
- C3. Géométrie différentielle.
- C4. Analyse sur les variétés.
- C5. Groupes algébriques, fonctions automorphes et groupes semi-simples.

D. Analyse.

- D1. Espaces vectoriels topologiques.
- D2. Algèbres d'opérateurs; représentations des groupes localement compacts.
- D3. Théorie spectrale.
- D4. Algèbres de fonctions; analyse de Fourier.
- D5. Théorie du potentiel; processus de Markov.
- D6. Probabilités, théorie de la mesure, intégration.
- D7. Fonctions analytiques d'une variable complexe.
- D8. Fonctions et espaces analytiques complexes.
- D9. Ensembles exceptionnels en analyse.
- D10. Analyse fonctionnelle et équations aux dérivées partielles linéaires.
- D11. Analyse fonctionnelle et équations aux dérivées partielles non linéaires.
- D12. Systèmes dynamiques et équations différentielles ordinaires.

E. Mathématiques appliquées.

- E1. Aspects mathématiques de la théorie quantique des champs.
- E2. Théorie de la relativité.

- E3. Problèmes mathématiques de la mécanique du continu.
- E4. Théorie de contrôle optimal.
- E5. Combinatoire et algèbre finie.
- E6. Statistique mathématique.
- E7. Problèmes mathématiques de la théorie de l'information, langage machine.
- E8. Analyse numérique.

F. Histoire et Enseignements.

- F1. Histoire.
- F2. Enseignements.

This was a tremendously drastic change to the way the sections were previously organised. If we look at the list in detail, there is a clear desire to show an underlying structure in mathematics, to make clear, through the congress, the fact that mathematics is shaped by a certain 'architecture'. That was precisely the goal of the Bourbaki programme, which in 1948 had published an article titled 'L'architecture des mathématiques'.

The explanation for this unexpected change in the international congresses is simple: Some of the renowned members of the Bourbaki group formed part of the organising committee. These included Jean Dieudonné, Laurent Schwartz, Jean-Pierre Serre and André Lichnerowicz (who was the leading figure in a reformation of the teaching of mathematics in 1960 and 1970). The president of the International Mathematical Union at the time, Henri Cartan, also belonged to the Bourbaki group.

THE LANGUAGE OF SCIENCE

Despite the organisers' best efforts, French was not the dominant language at the Nice congress. Of the 16 plenary conferences, only one used French instead of English – that of the Russian Lev Semenovich Pontryagin. Of the 242 specialised expository conferences, 190 were in English and only 49 in French (the remaining three were in German). Thirty-eight years earlier, at the Zurich congress in 1932, of a total of 268 lectures, 112 were in French, 97 in German, 33 in English and 26 in Italian. The debate on the language of science had been decided by the economic changes after World War II.

We conclude the story of the Nice congress at the beginning – the awarding of the Fields medals. There were four winners: the English mathematician Alan Baker from Cambridge University for his work on transcendent numbers; the Japanese mathematician Heisuke Hironaka from Harvard University for his result on the resolution of singularities in algebraic varieties, which “is not completely platonic, on the contrary, it is a powerful tool”; the Soviet mathematician Sergéi Novikov from Moscow State University, the “great originality and very powerful technique” of whose work on algebraic geometry and topology was commended; and the American mathematician John G. Thompson from Chicago University for his contributions to the classification of simple finite groups.



Winners of the Fields Medals in 1970: from left to right, Alan Baker, Heisuke Hironaka, Sergéi Novikov and John G. Thompson (photo: Harald Hanche-Olsen).

Sergei Novikov’s medal was not collected, as the government of the USSR did not give him permission to attend the congress. Twenty-two Soviet mathematicians were prevented from participating for the same reason. Both the organisation and many participants regretted this fact.

Henri Cartan personally presented the medal to Novikov, as the president of the International Mathematical Union, at a private dinner, when Cartan attended a meeting in Moscow in honour of 80 years of the Vinogradov theorem.

Journey to the centre of Canadian counter-culture

The University of British Colombia held the international congress in 1974. After the upheaval of the usual model to suit the Bourbakist one at the previous congress, the scientific programme reverted to its traditional system, albeit with a detailed list

of scientific sections. Instead of the branched-tree structure used in Nice, a linear one was used:

1. Mathematical logic and fundamentals of mathematics.
2. Algebra.
3. Theory of numbers.
4. Algebraic geometry.
5. Algebraic groups and discrete subgroups.
6. Geometry.
7. Algebraic and differential topology.
8. Differential geometry and analysis in varieties.
9. General topology and real and functional analysis.
10. Algebras of operators, harmonic analysis and representation of groups.
11. Mathematical probability and statistics.
12. Complex analysis.
13. Equations in partial derivatives.
14. Ordinary differential equations and dynamic systems.
15. Control theory and related optimisation problems.
16. Mathematical physics and mechanics.
17. Numerical mathematics.
18. Discrete mathematics and computation theory.
19. Applied statistics and mathematics in social and biological sciences.
20. History and education.

This plan included 17 plenary conferences, 157 invited lecturers associated with one of the sections and 565 short talks within the sections. A certain commitment to the Nice model is noticeable in the significant increase in the number of invited lecturers in the sections (there were only 64 in Moscow). The attendance of the congress did not reach the heights of that of Moscow, but even so a total of 3,120 mathematicians took part (of whom 40% were Americans) and the congress was still a success.

The Fields Medals were of note. Only two were awarded, although the commission responsible for the awards (formed, of course, by the highest calibre of mathematicians) could have given four. The most plausible explanation is heated internal disputes in the commission, as there was no lack of candidates or significant results.

Membership by Countries

Algeria	2	Kuwait	4
Argentina	3	Lebanon	1
Australia	44	Libya	1
Austria	4	Malaysia	2
Belgium	14	Mexico	20
Brazil	8	Netherlands	39
Bulgaria	5	New Zealand	15
Canada	514	Niger	1
Chile	9	Nigeria	16
China, Republic of	11	Norway	16
Columbia	1	Pakistan	7
Congo	1	Philippines	2
Costa Rica	1	Poland	18
Cuba	1	Portugal	3
Czechoslovakia	4	Puerto Rico	2
Denmark	19	Rhodesia	1
Egypt	2	Romania	5
England	181	Saudi Arabia	1
Finland	11	Scotland	27
France	245	Senegal	10
Germany, Democratic Republic of	11	Sierra Leone	1
Germany, Federal Republic of	146	Singapore	2
Greece	7	South Africa, Republic of	9
Guatemala	2	Spain	11
Hong Kong	3	Sweden	17
Hungary	24	Switzerland	28
India	26	Tunisia	3
Iran	11	Turkey	2
Iraq	4	Uganda	1
Ireland, Northern	1	Uruguay	1
Ireland, Republic of	7	U. S. A.	1274
Israel	20	U. S. S. R.	50
Italy	45	Venezuela	2
Ivory Coast	2	Vietnam, Democratic Republic	4
Japan	114	Wales	6
Kenya	3	Yugoslavia	10
		Zaire, Republic of	3

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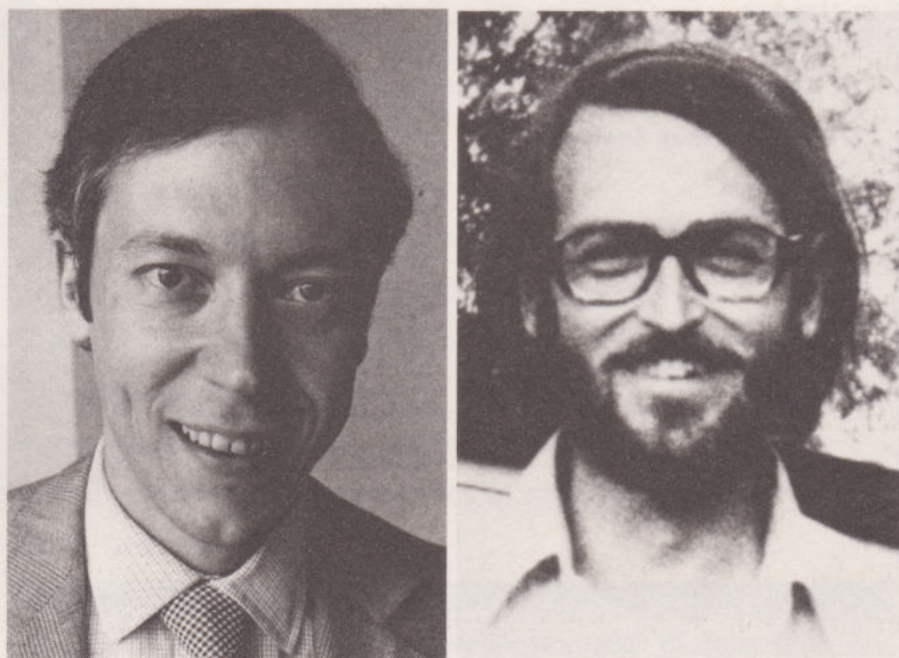
Participants by country at the Vancouver conferences in 1974.

The winners were the Italian mathematician Enrico Bombieri from the University of Pisa for his achievements in number theory, univalent functions, several complex variables, partial differential equations and algebraic geometry, areas in which “his versatility and strength have combined to create many original patterns of ideas”; and the English mathematician David B. Mumford from Harvard University for his “tremendously successful multi-pronged attack on problems of the existence and structure of varieties of moduli”.

An incident occurred during the award ceremony, although few people noticed in the Queen Elizabeth theatre where the ceremony was held. Nobody had been allotted to take the medals out of the safe at the bank where they had been deposited

a few days before for security reasons. The ceremony was delayed while the organisers rushed to collect them under a police escort.

Explicit tensions with the USSR regarding the participation of the Soviet mathematicians showed their face at this congress, more specifically in the choice of the invited speakers, both in the plenary conferences and the sections. The Soviet representative in the commission responsible for the selection, S.V. Jablonskii, explained that the Soviet scientific institutions were more capable than any other of judging and, therefore, choosing the Soviet mathematicians who should be invited to the congress. This principle of working eliminated the role of any international commission in the configuration of the scientific programme.



In 1974 only two Fields Medals were awarded, to the Italian Enrico Bombieri (left) and the Briton David Mumford.

Jablonskii's criticism went as far as considering that: "Certain mathematicians from the Soviet Union have been selected for Vancouver, who either lack scientific merit or have already been invited to previous congresses, while, on the other hand, mathematicians with new and interesting results, highly recommended by the main Soviet specialists, have been left out."

This posture was supported by the influential Soviet mathematician Lev Semenovich Pontryagin, who was a member of the executive committee of the International Mathematical Union. In the end it was decided to maintain the initial

list of invited speakers, which meant that of the 42 invited Soviet mathematicians, only 22 attended.

These tensions were behind the scenes, although they could be felt in the atmosphere of the congress. However, the overriding memory of the environment was of something else: at the beginning of the 1970s, Vancouver was the counter-cultural capital of Canada. The world was changing and the social conventions on behaviour, personal relationships, fashion and political attitudes were collapsing, while many other things were bubbling under the surface, such as pacifism, environmentalism, more sexual freedom, the end of reverence for authority, informal dress..., and universities were a melting pot of these cultural movements.

The congress was held on the peninsula that is home to the University of British Columbia, on the Strait of Georgia. The conferences were given in the university buildings, and the participants were housed in the student residences on the campus itself. This, together with the mild weather at the end of August, created a feeling of camaraderie which, along with the city's counter-cultural and hippy movements, gave the congress a very unusual tone. This atmosphere can be appreciated in all the images, with groups debating on the grass, meetings with beer and music, attendees whose appearance would have meant they could not enter the university a few years earlier; even at the formal Fields Medal award ceremony there was a hint of the *laissez-faire* attitude.

To top it all off, on the coasts of the peninsula where the university is there are beautiful beaches which are washed by the waters of the Strait of Georgia. One was Wreck Beach, famous at the time for being one of Canada's best nudist beaches. This was another attraction at the congress, which undoubtedly served to disseminate the revolution of customs of the period, at least among the mathematicians.

At the time there was profound incomprehension and distrust among the generations. Academic life, previously a bastion of strictness, learning and formalities, had now changed. The opening address of the renowned Canadian geometrician Harold Scott MacDonald Coxeter, presiding over the congress, was indicative of the fears of the older generation.

"The present generation has been engulfed by a wave of anti-intellectualism, with the result that most universities are short of students. Young people find that the problem of looking for a job is not facilitated by a university education. The idea of 'art for art's sake' is less prevalent than it used to be."

Interference

The three following congresses, Helsinki in 1978, Warsaw in 1983 and Berkeley in 1986, were marked by the vicissitudes of history, more specifically by the direct effects of the Cold War and realpolitik.

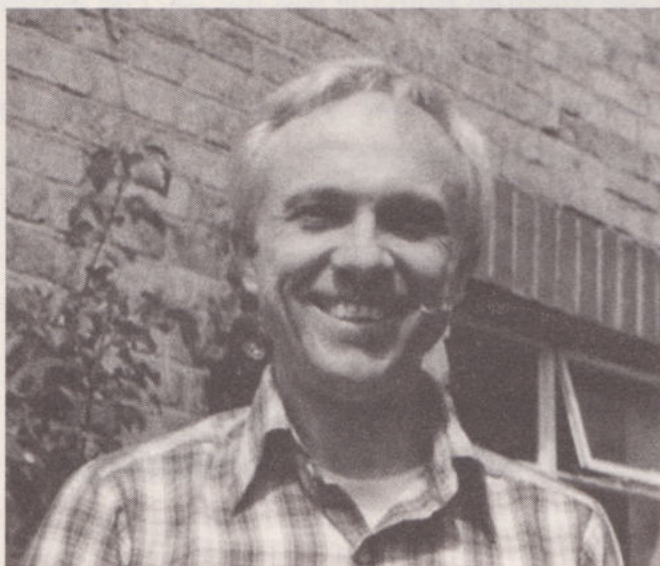
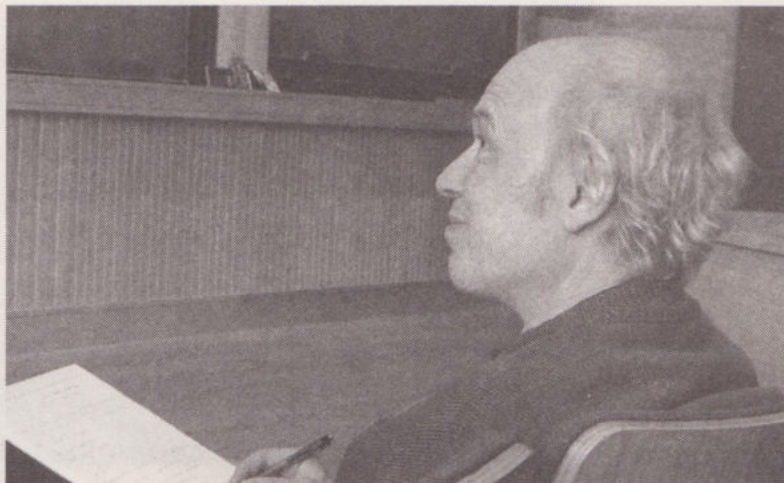
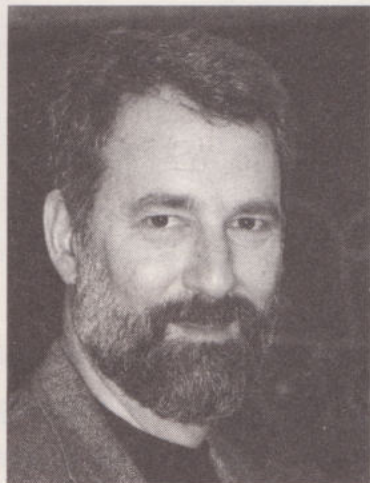
The Helsinki congress in 1978 was a model congress in terms of its preparation and content. It was not in vain that the main organiser was the Finnish mathematician Olli Lehto, who from 1983 to 1990, would be the secretary – and soul – of the International Mathematical Union. The overall flavour of the opening ceremony was somewhat nationalistic: music by Finnish composer Jean Sibelius in the impressive conference building Finlandia Hall, designed by the famous Finnish architect Alvar Aalto.

This time there was no pettiness and there were four Fields Medals: the Belgian mathematician Pierre René Deligne from the Institute of High Scientific Studies (IHES) for his work on “surprising relationships between the cohomological structure of algebraic varieties over the complex numbers, and the Diophantine structure of algebraic varieties over finite fields”; US mathematician Charles L. Fefferman from Princeton University, whose great contributions to the “vitality of analysis” were commended; Soviet mathematician Gregori Aleksandrovich Margulis, from the Moscow Institute for Information Transmission Problems, whose work “belongs to combinatorics, differential geometry, ergodic theory, dynamical systems, and Lie groups”; and the American Daniel G. Quillen, from the Massachusetts Institute of Technology (MIT), whose “contributions to mathematics are outstanding in their inventiveness, conceptual richness and technical virtuosity”.

As at the Nice congress in 1970, the Fields Medallist Gregory A. Margulis did not collect the medal which had been awarded to him. He could not, since he did not receive permission from the Soviet authorities to attend the congress. This led to great dejection at the congress and to Jacques Tits, the Belgian mathematician responsible for the ‘laudatio’ to the congress for the work of Margulis (the official explanation of the reasons for which the Fields medal is awarded), to say:

“I would like to conclude my report with a non-mathematical comment. This may not be the time to start a controversy. Despite this, I cannot but express my deep disappointment – no doubt shared by many people here – in the absence of Margulis from this ceremony. In view of the symbolic meaning of this city of Helsinki, I had indeed grounds to hope that I would have a chance at last to meet a mathematician whom I know only through his work and for whom I have the greatest respect and admiration.”

Tits was alluding to the agreements recently signed in Helsinki in 1975 on security and cooperation in Europe, where “respect for human rights and fundamental liberties” was established as one of the principle points.



The “Fields” of 1978. From top to bottom and left to right: Charles Fefferman, Pierre Deligne, Daniel Quillen and Gregori A. Margulis (photo: George M. Bergman).

Margulis’ absence was more evidence of discrepancies over the selection of speakers from the USSR invited to the congress, as had happened at the previous congress in Vancouver. Lev Semenovich Pontryagin again expressed “the profound dissatisfaction” of the National Committee of Mathematics of the USSR with the mathematicians chosen to speak and for the Fields Medal. The effect was the absence of, as well as Margulis, three of the Soviet plenary speakers and 11 of those invited to speak at other times. Thirty years later, Margulis explained to me what happened with the medal: “Curiously, as I was not able to attend the congress in Helsinki, in

1979 I was authorised to visit the West for the first time, on a three-month visit to Bonn. There Jacques Tits visited me, and he presented the medal to me in a small ceremony.”

APARTHEID

The organisers of the congress also had difficulty with the Finnish political authorities, who did not allow the participation of South African mathematicians because of apartheid. Everything was resolved positively, returning to the principle of the free circulation of scientists.

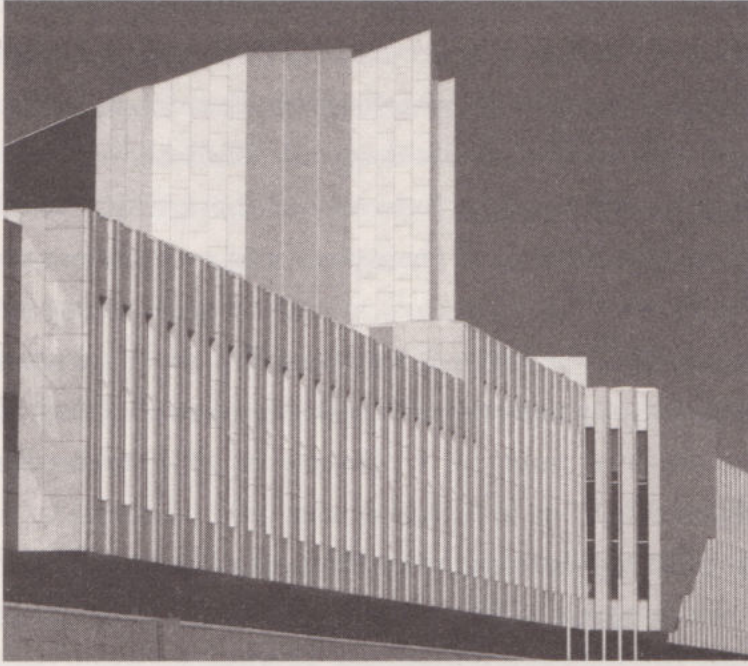
Mathematics stands out from the other sciences for, among many other things, its often passionate interest in its history, which covers everything from the fledgling mathematician to the established professional. One of the reasons may lie in the fact that the problems, the work and the solutions from the past live on. The lecture that attracted the most listeners of all those programmed at the Helsinki congress was the plenary session by French mathematician André Weil, titled: “The history of mathematics: Why and how?” Weil started by saying:

“My first point will be an obvious one. In contrast with some sciences the whole history of which consists of the personal recollections of a few of our contemporaries, mathematics not only has a history, but it has a long one, which has been written about at least since Eudemos (a pupil of Aristotle). Thus the question ‘why?’ is perhaps superfluous, or would be better formulated as ‘for whom?’ ”

Following on from this point of view, Weil cited the German mathematician Gottfried Leibniz:

“Its use is not simply that history can give everyone what they need and others can work for a similar payment, but also to promote the art of discovery and its known methods through illustrious examples.”

The prestige of the speaker and the attractive subject meant that more than 3,000 people gathered, many of whom had to follow the lecture via televisions located in the corridors of Finlandia Hall.



Finlandia Hall in Helsinki.

THE POLISH BOAT

The standard of living in Helsinki in 1978 greatly increased the cost of attending the congress. The Polish delegation found a solution to the problem by hiring a boat to travel to Helsinki; after docking in the port they used it as accommodation and a restaurant. There are great memories of the night meetings on the vessel, but there are no photos.

Deciding the location of the next congress was a tumultuous affair because four countries offered their services: Argentina, the Federal Republic of Germany, Israel and Poland. The first two had serious political objections (the military dictatorship in Argentina and, in the case of West Germany, the Soviet veto and memories of the Holocaust). The discussion focussed on Israel and Poland, two countries with a magnificent scientific curriculum in mathematics. In the end Poland was selected, based on its greater political stability than that of the Middle East. As we will see, it was the right argument but the wrong decision.

The congress was to be held in Warsaw in August 1982. As had already happened at previous congresses, problems occurred due to the resistance of the Soviet

Union to having an international committee choose Soviet speakers. During the discussions there were very strong accusations from both sides (from anti-Soviet activities to Zionist propaganda), which culminated in the threat of the Soviet representatives to withdraw from the congress. The intervention of the ultimate Soviet authority, the Academy of Sciences of the USSR, which prohibited a unilateral withdrawal, put things in their place and an agreement was reached. So far, so good.

But throughout 1981, the political, economic and social situation in Poland was worsening, as the Solidarity trade union increased pressure on the Polish regime. The situation reached breaking point on 13 December 1981, when General Jaruzelski declared martial law.

In February 1982, the International Mathematical Union discussed whether or not to keep the congress in Warsaw. Apart from the general political situation, it was known that the Polish Mathematical Society had been suspended and that mathematicians were arrested and imprisoned. Great controversy arose in the mathematical community over whether to seek an alternative venue (Belgium had been offered).

There was no unanimity within Poland either, although the organisers of the congress, and the large majority of Polish mathematicians, were inclined to go ahead with the congress; the need to maintain contact with foreign colleagues was argued. (Nor was there, or is there, within Poland or elsewhere, a unanimous opinion on whether or not Jaruzelski's coup d'état sought to avoid Soviet military intervention or maintain the Polish communist regime.)

A few months later, in April 1982, the International Mathematical Union decided to postpone it by one year, until August 1983, in view of the Polish government's ambiguous support for the congress, and in view of the lack of mathematicians who had signed up to participate in the congress,

Throughout 1982, the situation was very confusing. Further controversy divided the international mathematical community once again over whether or not to hold the congress in Poland at all. The Polish organisers strongly believed that it should go ahead. Meanwhile, the Solidarity union was declared illegal. The Swedish mathematician Lennart Carleson, who we have already crossed paths with at the Moscow congress, as president of the International Mathematical Union at the time, personally communicated the conditions for holding the congress in Warsaw to the Polish government. There had to be: free circulation for all scientists; re-establishment of the Polish Mathematical Society; normal conditions

for transport and communication; and the release of those mathematicians being held in custody.

Finally, it was decided that the congress would be held in Warsaw in August 1983. As the date approached, Lech Walesa, the leader of Solidarity, was freed, and martial law was lifted. But, once again, the controversy polarised the mathematicians. They either had to attend the congress to support the government's opponents or not attend in protest against the government. For mathematicians from countries in the communist bloc, a congress in Warsaw was a wonderful opportunity (it was easier for them to get permission to go). The Westerners, meanwhile, were concerned about their personal security.

The historian Aleksander Gieysztor, president of the Polish Academy of Sciences, remembered part of Poland's tortured history at the congress' opening ceremony:

"The months of August and September encompass two important dates in the history of this country. Thirty-nine years ago on the 1st of August, the Warsaw Uprising began and September 1st 1939 was the first day of World War II. Both these months are times of national remembrance, of reflection upon the history of our country.

"During World War II, the Polish scientific community was decimated. In particular, well over half of the actively working Polish mathematicians lost their lives. Many others found themselves in various countries all over the world. Universities, libraries and printing presses in Poland were largely destroyed. The educational system of the country was in ruins and scientific activity was disrupted.

"The fact that this Congress is being held in Warsaw in 1983, thirty-eight years after the war, is evidence of the reconstruction of Polish science both in organisation and in substance. In particular, it is a proof of the renaissance and expansion of the Polish mathematical community."

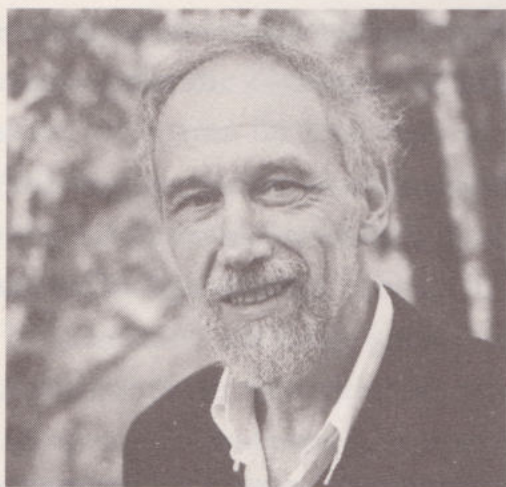
The Polish mathematicians Kazimierz Kuratowski and Stefan Banach were not forgotten either, and nor was the magazine *Fundamenta Mathematicae*, which was, when it was founded in 1920, the first mathematics magazine to specialise in one field. This series of homages was crowned with the election of Władysław Orlicz, the great mentor of many generations of Polish mathematicians, as the honorary president of the congress.



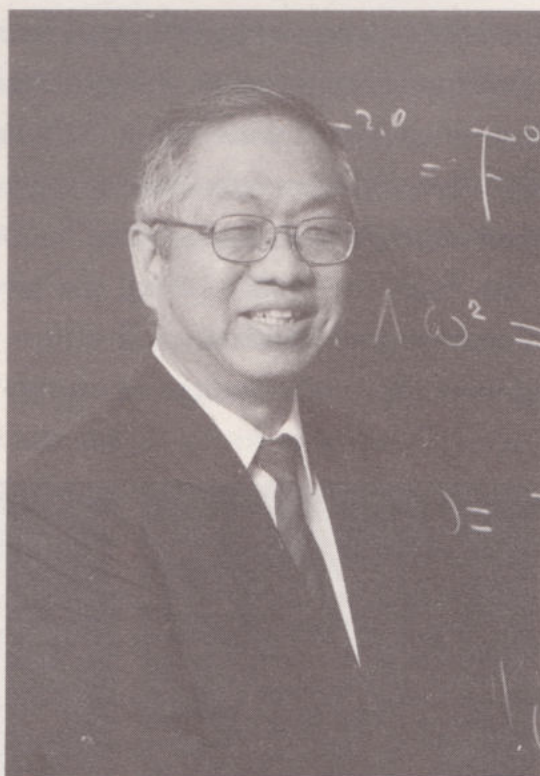
*Top: Kazimierz Kuratowski (left) and Władysław Orlicz.
Bottom: Bust of Stefan Banach in Krakow.*

Being in Poland, there could not be an opening ceremony without music – a lot of music – by Gorczycki, by Pacius, by Nowowiejski, by Palestrina, by Pekieli, by Wallek-Walewski and by Moniuszko, among others.

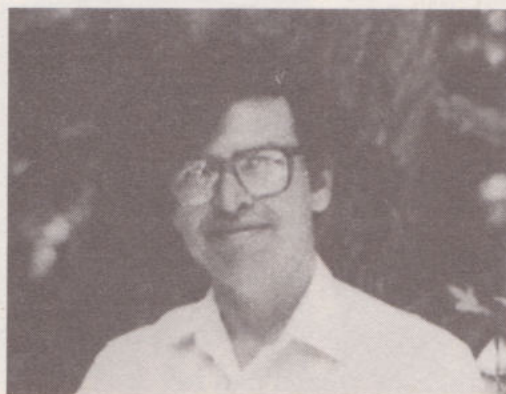
As for the medals, this time there were no surprises, as the names of the winners had already been announced one year before on the date initially planned for the congress. There was a surprise in the number of medals awarded, and the old trick of not awarding a fourth, just as in Vancouver, was back. Three were presented: the French mathematician Alain Connes from the Institute of High Scientific Studies for his “breathtaking achievements, exceeding experts’ expectations”, in the theory of algebra of operators and applications of differential geometry; to the US mathematician William Thurston from Princeton University for his “fantastic intuition and geometric vision” applied to the study of topology in dimensions two and three and the interrelation between analysis, topology and geometry; and to the Chinese mathematician Shin-Tung Yau from the Princeton Institute for Advanced Study for his contributions to global differential geometry, equations on derived partials and algebraic geometry.



Alain Connes.



Shin-Tung Yau.



William Thurston (photo: George M. Bergman).

The Nevanlinna prize was awarded for the first time to the US computer scientist Robert Tarjan from Stanford University for his work on “design and analysis of algorithms”.

The 1982-1983 congress in Warsaw (it was decided to describe it in this way, in order to maintain the four-yearly cycle) was a normal congress compared to the previous one in Helsinki and the next one in Berkeley, both in terms of attendance (3,038 in Helsinki, 2,450 in Warsaw and 3,586 in Berkeley) and in terms of plenary conferences, invited speakers in the sections and announcements: 17/119/500 in Helsinki, 16/129/680 in Warsaw and 19/148/731 in Berkeley. They are even similar in the number of invitees who did not attend: 14 in Helsinki, 35 in Warsaw and 15 in Berkeley. The difference is in the fact that both in Helsinki and in Berkeley the Soviets were the absentees, while in Warsaw it was the Westerners, mainly Americans. There was an additional reason that contributed to the lack of attendance of American mathematicians at the Warsaw congress, the fact that the US administration did not authorise the use of public funds to travel to Poland, despite the US National Academy of Sciences' desire to the contrary. These absences were compensated, in a certain way, by the high attendance of Soviet mathematicians and those from other communist countries. The feeling of insecurity that was still being transmitted by the Polish political situation did affect the number of partners attending – just 150 in Warsaw compared with 900 in Helsinki.

In summary, what was close to becoming a disaster ended up, given all the circumstances which surrounded it, a success. There was a lot to be grateful for: the Poles for their hosting, the level of science and the lack of incidents; the International Mathematical Union for having saved the continuity of the congresses; those who wanted to express their solidarity with the Polish attendees for having been able to do so; for the absence of governmental manipulation, despite many making a public rejection of the repressive measures in their lectures. *All's well that ends well.*

Commemorating the fact that 1986 was the 50th anniversary of the first Fields Medal award, the congress held at the University of Berkeley named the Finnish mathematician Lars V. Ahlfors, one of the first two medallists, as honorary president. Ahlfors remembered the occasion in Oslo in 1936:

“At that time the circumstances were quite different; the idea of the medals had been approved in Zurich in 1932, but there had been no publicity about it. When I arrived in Oslo I did not know that the medal had become a real-

ity, and if I had known it I would not have considered myself a candidate. As a matter of fact, I had not been told anything officially until I entered the room where the opening ceremony would take place, but there I was shown to a seat somewhere in the front, and I may have had my suspicions. Well, I had more than that. I had been warned beforehand by somebody who congratulated me a day before by mistake. But up to that point it had been a secret at least officially, even to myself. There was no tradition to go by and no protocol to follow.”

Lars V. Ahlfors was also responsible for presenting the medals. For the third time since a maximum of four medals was first allowed in 1966, fewer than four were granted. In this case there were three, which went to the British mathematician Simon K. Donaldson from Oxford University for his work on 4-dimensional varieties “which have surprised the mathematical world and which implicate the existence of exotic four-dimensional spaces”; the German mathematician Gerd Faltings from Oxford University for his proof of the Mordel conjecture, “one of the great moments in mathematics”; and US mathematician Michael H. Freedman from the University of California in San Diego for his proof of the Poincaré conjecture for four dimensions.

Ahlfors also awarded the Nevanlinna prize to computer scientist Leslie Valiant from Harvard University for his contributions “to the young and vigorous tree” of theoretical computation.



Simon K. Donaldson, Gerd Faltings and Michael H. Freedman.

THE FIRST BOOK ON THE CONGRESSES

The first book on the history of the international congresses, *International Mathematical Congresses: An Illustrated History 1893–1986*, written by the mathematicians Donald J. Albers and Gerald L. Alexanderson and the historian Constance Reid (well known for her biography of David Hilbert) was presented at the Berkeley 1986 congress. The book was a success, but the circumstances surrounding the Warsaw conference, which we have explained in detail, was very controversial. Therefore the publisher, the prestigious Springer Verlag from Berlin, had to withdraw the book and publish a revised edition.

The book can be freely accessed in pdf form on the web site of the International Mathematical Union (see the bibliography).

Now and again some of the official speeches actually have content and even substance. This was the case with the speech given during the opening ceremony of the Berkeley congress by Richard Johnson, nuclear physicist and scientific advisor to President Ronald Reagan (this was an institution created under the presidency of Dwight D. Eisenhower in response to the USSR's advantage in the space race). Johnson explained:

“We have a serious problem in the United States. Increasingly fewer young people are studying science, mathematics, engineering and technology; fewer people seek advanced levels in these disciplines, and fewer youths choose careers in scientific and technological fields.

“You have the rare talent to fully appreciate the depth and power of your subject. But, with ability and talent come obligations and responsibilities, especially to those not so blessed. What you do in your laboratories, at your computer terminals, and in your studies is remarkable; indeed, it has changed the course of the world. But you must do more. It is no longer enough to be practitioners of science. You must also become citizens of science, leaders of a uniquely gifted community, bound by the common goal of assuring that the progress you have made and the improvements you have wrought for humanity will continue. Especially, you must help to assure that there is an expanding, well-qualified generation of scientists, engineers, and technologists to succeed you.”

There were new concerns and a new public opinion of science and scientists, which was reflected by the administration.

The Berkeley congress in 1986 was attended for the first time by the People's Republic of China, which, after the devastating Cultural Revolution, had decided to rejoin many international activities, among them the International Congresses of Mathematicians. A major problem arose with the Republic of China in Taiwan's membership of the International Mathematics Union. It was not allowed by Red China based on the principle of 'one China'. The solution was a change of statutes in the International Mathematical Union itself in order to eliminate the adjective 'national' from the delegations of the countries, and therefore allow two Chinas. There were 30 mathematicians from the People's Republic and 15 from Taiwan.

MEMBERSHIP BY COUNTRY

Algeria 3	Japan 176
Argentina 3	Jordan 1
Australia 20	Kenya 2
Austria 8	Kuwait 9
Bahrain 2	Lebanon 1
Bangladesh 1	Luxembourg 1
Belgium 20	Malaysia 3
Botswana 1	Mexico 10
Brazil 20	Netherlands 27
Bulgaria 4	New Zealand 10
Cameroon 3	Nicaragua 2
Canada 167	Nigeria 8
Chile 3	Northern Ireland 2
China-Taiwan 17	Norway 16
Colombia 2	Oman 1
Costa Rica 1	People's Republic of China 30
Cuba 3	Philippines 2
Czechoslovakia 6	Poland 17
Denmark 14	Portugal 7
Egypt 2	Romania 3
England 94	Saudi Arabia 6
Ethiopia 1	Scotland 12
Federal Republic of Germany 119	Senegal 2
Finland 19	Singapore 5
France 117	South Africa 11
German Democratic Republic 4	South Korea 6
Ghana 1	Spain 18
Greece 6	Sweden 29
Guatemala 2	Switzerland 33
Hong Kong 16	Thailand 1
Hungary 12	Trinidad 1
Iceland 6	Turkey 4
India 21	U.S.A. 2324
Indonesia 1	U.S.S.R. 57
Iran 31	U.S.V.I. 1
Iraq 4	Uruguay 2
Ireland 4	Venezuela 3
Israel 34	Vietnam 3
Italy 47	Wales 5
Ivory Coast 9	West Indies 1
Jamaica 1	Yugoslavia 10

Total 3711

Participants by country in 1986.

One American peculiarity of the congress was its organisation, which was the responsibility of a corporation created specifically for that purpose, with a CEO, manager, etc., as if it were a commercial company and which, in a very practical way, classified the bodies and companies that collaborated financially into seven categories, according to the magnitude of their contribution.

There was another peculiarity in the musical programme. It was the only international congress to include a jazz concert, with 1,500 attendees! The counterpoint was the cultural visit, to the Cow Palace arena in San Francisco (which has hosted concerts by Elvis Presley, the Beatles, the Rolling Stones and the Doors, among others) to watch a rodeo and enjoy a Western-style barbecue.

Chapter 7

From the *Urbe* to the Orb

From the point of view of historical eras, the 20th century could be considered to have started late – in 1914. The same reasoning would also lead us to conclude that it ended early. Between the autumn of 1989, in which the Berlin Wall and most of the political regimes of Eastern Europe fell, and the autumn of 1991, in which the USSR imploded, the world experienced, not for the first time in the century, a profound change that would have lasting consequences. This is why we could consider the 20th century to have ended in early 1990s.

Among the many consequences of these changes, the most notable was the readjustment of the economy and, in the long term, the realignment of the centre of social and scientific gravity that shifted beyond what we call the West and into the rest of the world, particularly towards the East. We can say that we moved from the *urbe*, a word that the Romans used to refer to the city of Rome, and which we could use to mean the West, to the ‘orb’, the whole world.

This period of opening up can be seen clearly in the international congresses. After the sequence of twenty congresses held in Europe and North America between 1897 and 1986, three of the following congresses were held in oriental cities: Kyoto in 1990, Beijing in 2002 and Hyderabad in 2010. That shows, at the same time, the strength and expectation of scientific development in this region. The eastward movement was not temporary, but has been continuous. The 2014 international congress is planned to be held in Seoul. Of the remaining three congresses, the one in Berlin in 1998 in some ways finally healed the many wounds of World War II; Madrid in 2006 included a European country that was a latecomer to the scientific world; and that old perennial, Zurich, filled a gap for the 1994 congress.

The West

Ninety-four years after the Heidelberg congress in 1904, Germany hosted an international congress of mathematicians once again. It had been a strange exception among the great powers of the mathematical world, each of which had organised

at least two congresses. However, World War II, the Holocaust and the partition of Germany, explains it all.

The congress was held in Berlin, a living incarnation of the passing of history, which went from being the capital of the Kaiser's empire, through the city divided by the Iron Curtain, to the capital of a newly united and democratic Germany. The same thing occurred to the honorary president of the congress, Friedrich Hirzebruch, who presided over the Union of German Mathematicians in 1961, when the Berlin Wall was raised and mathematical society was split in two, and then did so again in 1990, when the country was unified – as were the mathematical societies. Attendance at the congress was high: 3,346 mathematicians from 98 countries, and a large number of mathematical contributions, a total of 1,759, broken down into 21 plenary conferences, 169 invited conferences in the sections, 1,098 short communications, 236 presentations of posters and 235 presentations of other types.

96

LIST OF PARTICIPANTS

PARTICIPANTS BY COUNTRY

(according to their mailing address)

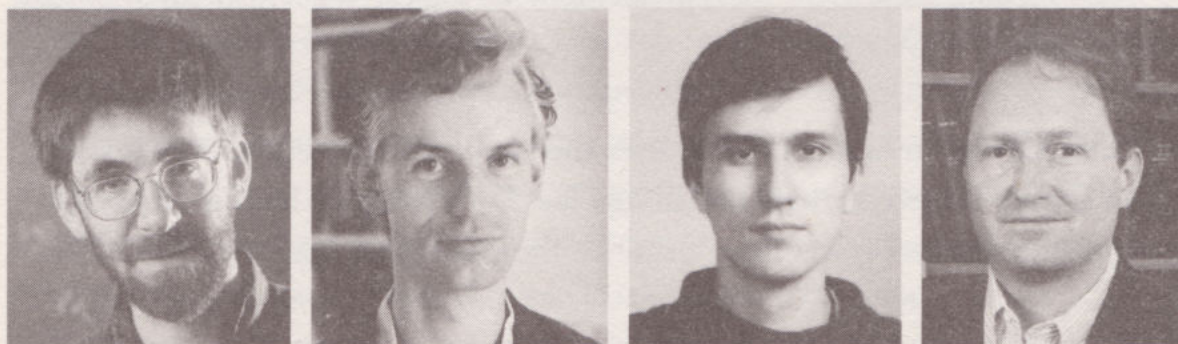
Algeria.....	5	Guatemala.....	4	Peru.....	1
Argentina.....	15	Hungary.....	25	Philippines.....	4
Armenia.....	4	Iceland.....	2	Poland.....	74
Australia.....	18	India.....	35	Portugal.....	18
Austria.....	17	Indonesia.....	1	Romania.....	43
Azerbaijan.....	1	Iran.....	24	Russian Federation.....	202
Bangladesh.....	1	Ireland.....	10	Senegal.....	1
Barbados.....	1	Israel.....	36	Singapore.....	4
Belarus.....	12	Italy.....	65	Slovakia.....	2
Belgium.....	9	Ivory Coast.....	5	Slovenia.....	1
Bolivia.....	1	Japan.....	205	South Africa.....	17
Bosnia and Herzegovina.....	2	Kazakhstan.....	7	Spain.....	44
Brazil.....	42	Kingdom of Saudi-Arabia.....	5	Sweden.....	37
Bulgaria.....	6	Korea.....	10	Switzerland.....	39
Burundi.....	1	Kuwait.....	3	Taiwan.....	11
Cameroon.....	2	Latvia.....	6	Tajikistan.....	8
Canada.....	45	Lebanon.....	1	Tanzania.....	1
Chile.....	11	Lesotho.....	1	Thailand.....	3
Colombia.....	4	Lithuania.....	6	Togo.....	1
Costa Rica.....	1	Luxembourg.....	2	Tunisia.....	6
Croatia.....	7	Macedonia.....	1	Turkey.....	8
Cuba.....	1	Malaysia.....	1	Turkmenistan.....	1
Cyprus.....	3	Mexico.....	15	USA.....	500
Czech Republic.....	11	Moldova.....	3	Ukraine.....	75
Denmark.....	11	Mongolia.....	2	United Arab Emirates.....	1
Egypt.....	7	Morocco.....	9	United Kingdom.....	88
Estonia.....	13	Mozambique.....	3	Uruguay.....	5
Finland.....	27	Netherlands.....	19	Uzbekistan.....	6
France.....	138	New Zealand.....	5	Venezuela.....	5
Georgia.....	13	Nigeria.....	4	Vietnam.....	5
Germany.....	1087	Norway.....	18	Yugoslavia.....	21
Ghana.....	1	P. R. China.....	52	Zimbabwe.....	2
Greece.....	10	Pakistan.....	1		

Participants by country in 1998.

Among the participants there were 353 from territories formerly belonging to the USSR, which was a very high number compared to the participation at the congresses in the 70s and 80s. Taking into account the extremely serious economic situation that those territories were left in, their attendance was possible only thanks to extraordinary financial assistance from the congress (nearly one million Deutschmark). The president of the Saint Petersburg Mathematical Society, Anatoly M. Vershik, publicly showed his gratitude, recalling the limitations of the past, even for the Fields medallists (Sergéi Novikov in Nice in 1970 and Gregori A. Margulis in Helsinki in 1978).

The prizegiving ceremony for the Fields Medals was carried out by the Russian mathematician Yuri Manin, the president of the Fields committee, who quoted the words of Georg Cantor: “Das Wesen der Mathematik liegt in ihrer Freiheit” – “The essence of mathematics is its freedom.”

Four medals were awarded: they were presented by Friedrich Hirzebruch to the South African mathematician Richard E. Borcherds from the universities of Cambridge and California in Berkeley for “his contributions to algebra, the theory of automorphic forms, and mathematical physics, including the proof of the Conway-Norton ‘moonshine’ conjecture”; the British mathematician William Timothy Gowers from Cambridge University for “his contributions to functional analysis and combinatorics, developing a new vision of infinite-dimensional geometry, including the solution of two of Banach’s problems”; to the Russian mathematician Maxim Kontsevich from the Institute of High Scientific Studies (IHES) for “his contributions to algebraic geometry, topology, and mathematical physics, including the proof of Witten’s conjecture”; and to US mathematician Curtis T. McMullen of Harvard University, for his contributions to “the theory of holomorphic dynamics and geometrization of three-manifolds, including the proof of Kra’s theta-function conjecture”.



Richard E. Borcherds, William Timothy Gowers, Maxim Kontsevich and Curtis T. McMullen.

The Nevanlinna Prize was also awarded to Peter Shor, of the AT&T Bell Laboratories, "for being the driving force behind quantum computing".

But the prize-giving ceremony did not end there. We have already explained the circumstances surrounding the proof of Fermat's last theorem by Andrew Wiles and the fact that he could not receive the Fields medal as he was older than 40. Yuri Manin remembered Fermat's formulation of the problem: "*Nullam in infinitum ultra quadratum potestatem in duas ejusdem nominis fas est dividere*", and presented Wiles with a commemorative plaque "for his sensational achievement". Paraphrasing Fermat himself he clarified that, unfortunately, the plaque was too small to write Wiles' proof on. The reports in the press speak of the "thunderous and continuous applause received by Wiles, which exceeded that of all the other winners".

The following day, Wiles gave a special lecture on 'Twenty Years of Number Theory', which attracted 2,300 listeners, and in which he explained:

"We begin with three problems considered by Fermat:

- "1. Which prime numbers can be written as the sum of two integer squares?
- "2. Is there a right-angled triangle with rational-length sides which has an area of one?
- "3. Do there exist solutions to the equation $x^n + y^n = z^n$ where n is greater than or equal to 3?

"The answer to the first question is that these are precisely the primes which are congruent to one modulo four. The answer to the second is no. The solution to these two problems were found by Fermat himself. The third problem of course needs no introduction."

After revising some of the main problems of the subject, Wiles made an important observation:

"One change in number theory over the last 20 years is that it has become an applied subject. Perhaps one should say that it has gone back to being an applied subject, as it was more than two thousand years ago."

As was to be expected, the congress was riddled with references to the atrocities perpetrated by the Nazis. From president of the Federal Republic, through the

president of the congress, to the Minister of Education and Research (who announced the creation of a prize for excellent young scientists, bearing the name of Emmy Noether, who had to flee from the country after the Nazis came to power), everyone remembered the dramatic events of the 30s and 40s and their terrible consequences on human lives and isolation (among them, the impossibility, for a long time, of holding a congress in Germany).

Friedrich Hirzebruch remembered many of the presidents of the Union of German Mathematicians:

“Alfred Pringsheim died in Zurich in 1941 at the age of 90 after having escaped from Germany. Edmund Landau lost his chair in Göttingen in 1934. Otto Blumenthal was deported to the concentration camp Theresienstadt, where he died in 1944. Hermann Weyl, president of our society in 1932, emigrated to the United States in 1933.”

He also remembered David Hilbert and his collaborators with the words written by Hermann Weyl in his master's obituary: “The Nazi storm broke, and those who had laid the plans and who taught there besides Hilbert were scattered over the earth.” Hirzebruch concluded by repeating Germany's obligatory motto since the end of the war: “We must teach the next generation not to forget.”

Two conferences on the effect of Nazi brutality on science were planned in a special section titled: “Victims, Oppressors, Activists, and Bystanders: Scientists' Response to Racial and Political Persecution”, led by Joel Lebowitz, of Rutgers University; and “Mathematics and Mathematicians in Nazi Germany: History and

MATHEMATIK UND ALLTAG

The first attempt to involve the non-mathematical public in an international congress came at the 1998 congress in Berlin. A programme was created called *Mathematik und Alltag* (“Mathematics and daily life”), which consisted of lectures – by, for example, writer and thinker Hans Magnus Enzensberger – a festival of mathematical videos and the projection of films and expositions, all aimed at the general public. The events were held in a strange Berlin institution called Urania, created in 1888 for the dissemination of science and culture and located in a curious navy blue, cubic building. More than 15,000 people visited Urania.

Memory”, by Herbert Mehrtens, of the Technische Hochschule in Braunschweig. The most intense and emotional of these initiatives was the ‘Terror and exile’ exhibition, organised by the Union of German Mathematicians (DMV), whose official advertisement is a very loyal reflection of the drama in the country:

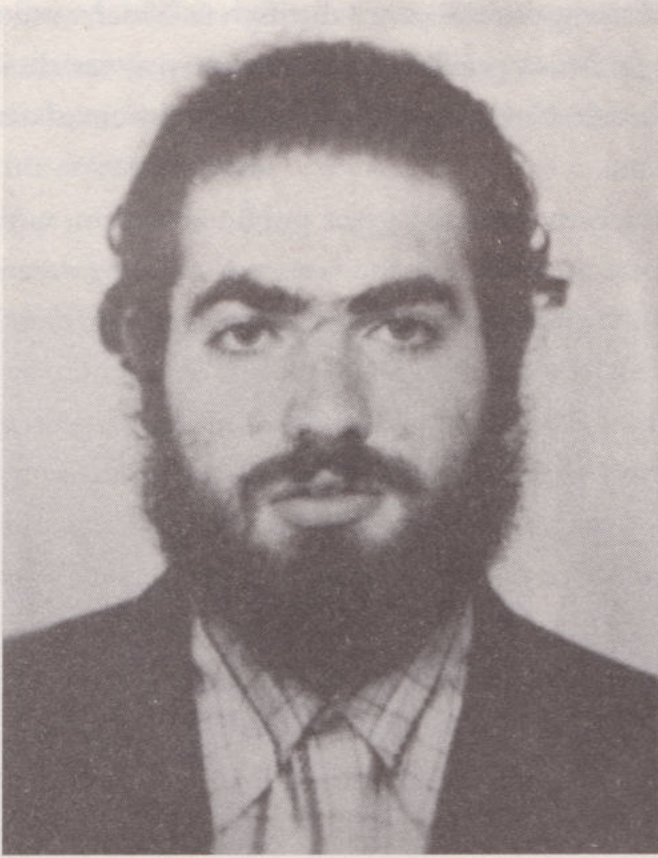
“In 1998 the ICM returns to Germany after an intermission of 94 years. This long interval covers the darkest period in German history. Therefore, the German Mathematical Society (DMV) wants to honour the memory of all those who suffered under the Nazi terror. The DMV does this in the form of an exhibition presenting the biographies of 53 mathematicians from Berlin who were victims of the Nazi regime between 1933 and 1945. The fate of this small group illustrates painfully well the personal sufferings and the destruction of scientific and cultural life; it also sheds light on the instruments of suppression and collaboration.”

Three of those 53 mathematicians, later based in the US, Great Britain and Australia, were invited by the Senate of Berlin to attend, and they told of their experiences of expulsion and emigration at a very emotional ceremony.

A BOOK ON THE INTERNATIONAL MATHEMATICAL UNION

At the 1998 congress in Berlin the book *Mathematics Without Borders: A History of the International Mathematical Union*, written by Finnish mathematician Olli Lehto (who we came across at the Helsinki congress in 1994) was officially presented. The book honoured the spirit of cooperation in the international mathematical community, recounting the troubled history of the institution.

Now let's turn our gaze to the other great Western congress of this period, the Madrid congress in 2006. An unprecedented event occurred there which defined the congress: the Russian mathematician Grigori Perelman, from Saint Petersburg, rejected the Fields Medal which had been awarded to him. It is not just that he did not attend the lecture to collect it: he simply refused to accept the prize. Nothing like that had ever happened. To date, Perelman's medal, with his name engraved around the edge, has not been collected by anybody, therefore it remains in the safe at the Royal Canadian Mint.



Grigori Perelman.

Months earlier, John Ball, a British mathematician from Oxford University and president of the International Mathematical Union in 2006, tried everything to convince Perelman to accept the prize even if he did not attend the congress, including long discussions and walks through Saint Petersburg; he was unsuccessful. Perelman had abandoned his position at the Steklov Institute in Saint Petersburg and, moreover, he had decided to abandon mathematics altogether.

Perelman was chosen as a winner of the Fields Medal for his solution to the Poincaré conjecture. Regarding the classification of three-dimensional surfaces, this was a problem proposed by Henri Poincaré in 1904. Generalised versions of the problem for higher dimensions had been resolved through the years by various Fields winners, Stephen Smale, William Thurston and Michael Freedman. The official information at the congress on the reasons for the choice of winner was more specific and, therefore, more cryptic: Grigori Perelman is awarded the medal for “his revolutionary insights into the analytical and geometric structure of the Ricci flow”.

The reasons for Perelman’s attitude have never been clarified, but they seem to be related to behaviour in the academic world which he considers reprehensible.

During the Madrid congress, the journalist Sylvia Nasar, author of John Nash's biography *A Beautiful Mind*, published a (controversial) article in the prestigious literary magazine *The New Yorker* in which she probed the circumstances surrounding the Perelman problem.

This unusual situation attracted great public attention, with more than 150 foreign press members accredited at the congress' opening ceremony, and a lot of talk on the street (the taxi drivers in Madrid, whenever they were given the address of the Palacio de Congresos, would ask "Has that Perelman guy turned up yet?"). This attention showed the danger of focussing on the eccentricity of Perelman's behaviour (and perhaps, therefore, of all mathematicians) instead of his more normal hobbies, such as going to the opera and taking walks. On the streets of the Russian city, I was able to verify that he is considered a kind of hero of modern times, a type of proud and honoured Don Quixote who snubs money and honours.

Fortunately, the rest of the prizes were presented without incident. There were three other Fields medals: the Russian mathematician Andrei Okounkov from Princeton University for "his contributions bridging probability, representation theory and algebraic geometry"; the Australian mathematician Terence Tao, from California University in Los Angeles, for "his contributions to partial differential equations, combinatorics, harmonic analysis and additive number theory"; and to the French-German mathematician Wendelin Werner from the University of Paris-Sud for "his contributions to the development of the geometry of two-dimensional Brownian motion, and conformal field theory". All the medals (except for Perelman's, of course) were presented by the King of Spain, who presided over the opening ceremony.

The scientific content of the congress included 20 plenary conferences, 169 invited conferences in the sections and some 1,000 short contributions and posters. The Poincaré problem and its resolution, the reason for awarding Perelman the medal, was ever-present at the congress. Attention was concentrated on the lecture titled 'The Poincaré Conjecture' by Richard Hamilton. (Hamilton was the creator of Ricci flow, the main tool in the proof of the conjecture – and an obvious candidate as he proved it himself). Another wonderful lecture by John W. Morgan, called "A Report on the Poincaré Conjecture", sought to make the problem accessible to those who were not specialists in the subject – and he seemed to achieve this, the slides can be seen at <http://www.mathunion.org/Videos/ICM2006/tars/morgan2006.pdf>).

For many reasons, the applications of mathematics played an important role at the congress. On the one hand, the Nevanlinna prize was awarded to the American

computer scientist Jon M. Kleinberg from Cornell University for “his deep, creative and insightful contributions to the mathematical theory of the global information environment”, which included many new applications aimed at optimising the operation of internet search engines.



King Juan Carlos I (centre) with the winners. From right to left, the three winners: Wendelin Werner, Andrei Okounkov and Terence Tao; next to the latter, on the far left, Jon M. Kleinberg, winner of the Nevanlinna Prize.

Continuing the applied theme, the Gauss prize for applied mathematics was awarded for the first time to the Japanese Kiyoshi Itô, at 90 years of age, for “having laid the foundations of the theory of stochastic differential equations and stochastic analysis” and considering that “Itô’s work is one of the major mathematical innovations of the 20th century and has found a wide range of applications outside of mathematics” in engineering, physics, biology, economics and finance.

To complete this focus on the applications of mathematics there was a public debate with important mathematicians, on the burning issue: are pure mathematics and applied mathematics drifting apart?

A new record was set at this congress. There were participants from 108 countries.

Participants by country*

Algeria	11	Hungary	21	Panama	1
Argentina	32	India	111	Paraguay	1
Armenia	1	Indonesia	2	Peru	5
Australia	23	Iran	38	Philippines	6
Austria	11	Ireland	8	Poland	32
Azerbaijan	11	Israel	34	Portugal	32
Bangladesh	1	Italy	81	P.R. China	33
Belarus	4	Ivory Coast	3	Puerto Rico	5
Belgium	12	Japan	128	Romania	22
Benin	1	Jordan	1	Russian Federation	75
Bolivia	1	Kazakhstan	4	Saudi Arabia	6
Bosnia & Herzegovina	3	Kenya	2	Senegal	2
Brazil	47	Kuwait	1	Serbia & Montenegro	15
Bulgary	2	Kyrgyzstan	1	Singapore	6
Burkina Faso	1	Latvia	3	South Africa	12
Cambodia	1	Lesotho	2	South Korea	61
Cameroon	4	Lithuania	1	Spain	1330
Canada	53	Luxembourg	1	Sudan	1
Chile	9	Madagascar	1	Sweden	21
Colombia	13	Malaysia	3	Switzerland	26
Croatia	6	Mauritania	1	Tadjikistan	1
Cuba	2	Mexico	57	Taiwan	24
Cyprus	1	Moldova	2	Tanzania	1
Czech Republic	7	Mongolia	2	Thailand	4
Denmark	9	Morocco	15	Tunisia	7
Ecuador	1	Mozambique	2	Turkey	16
Egypt	6	Nederlandse Antillen	1	Turkish Cyprus	2
Estonia	6	Nepal	1	Uganda	1
Falkland Islands	1	Netherlands	25	Ukraine	8
Finland	26	New Zealand	3	United Arab Emirates	4
France	122	Nicaragua	1	United Kingdom	103
Georgia	4	Nigeria	4	USA	383
Germany	120	North Korea	3	Uruguay	8
Greece	15	Norway	16	Uzbekistan	1
Guatemala	1	Pakistan	4	Venezuela	11
Hong-Kong	1	Palestine	1	Vietnam	6

* According to their mailing addresses.

*Participants by country in 2006.***MATHEMATICIANS DO NOT OWN MATHEMATICS**

This interesting statement is by John Ball, the president of the International Mathematical Union. In this spirit, the European Mathematical Society organised a debate titled "Should mathematicians care about communicating to broad audiences?" during the congress. The conclusions indicate the need to make everyone see the usefulness of mathematics.

The cultural content of the Madrid congress was varied. There was an exhibition of 16th-century mathematical texts in the Library of the Complutense at the University of Madrid; an international fractal art competition (the panel of judges was led by Benoît Mandelbrot, a great disseminator of and specialist in fractals); the Japanese sculptor Keizo Ushio sculpted an image of the infinity symbol on a granite rock weighing several tonnes live on the street during the ten days of the congress; a facsimile edition of three of Archimedes' great works (*On the Sphere and the Cylinder*, *On the Measurement of the Circle* and *The Squaring of the Parabola*) was made, based on manuscripts housed at the El Escorial Monastery and given as an official gift to the Fields medallists and other winners.



Keizo Ushio's sculpture for the 2006 congress in Madrid.

The high point of the Madrid congress cultural programme was the *Life of Numbers* exhibition, organised at the National Library. It was designed for the general public and sought to reflect the relationship between man and numbers throughout history, by means of a multiplicity of cultural objects from all around the world. The centrepiece of the exhibition was the *Codex Vigilanus*, a magnificent manuscript from the tenth century conserved at the El Escorial Monastery where the complete

list of Indo-Arabic numbers from 1 to 9 appeared for the first time in history. In the introduction to the exhibition catalogue (which could be better described as a treatise on the history of numbers and man) it is explained that:

“Numbers may be like the gods. No respectable civilisation or culture does not count them among their achievements, although sometimes it is not clear if they are to be classified as intellectual, magical or, simply, practical achievements. Like the gods, numbers change their name and iconography according to whether they were born of a southern, eastern or western civilisation. But at the same time, perhaps numbers have very little to do with the gods as, unlike them, numbers, although differently dressed, are essentially the same, be they the children of a culture from this side of the ocean or from overseas, from this sea, another or from the great beyond. And, having said this, numbers exist, something which perhaps could not be said of some gods.”

THE WORK OF THE MATHEMATICIAN

John Ball praised the ‘community work’ carried out by the mathematician:

“Mathematics has a strong record of service, freely given. We see this in the time and care spent in the refereeing of papers and other forms of peer review. We see it in the running of mathematical societies and journals, in the provision of free mathematical software and teaching resources, and in the various projects worldwide to improve electronic access to the mathematical literature, old and new. We see it in the nurturing of students beyond the call of duty.”

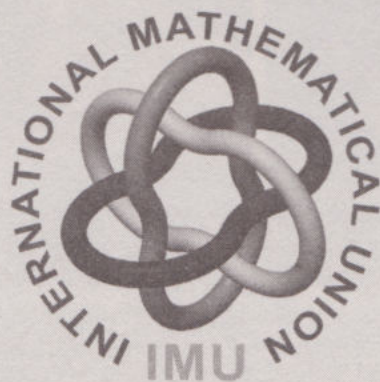
At the end of the congress, the Hungarian mathematician László Lovász, president of the International Mathematical Union for the period from 2007 to 2010, returned to the recurring theme of the opportunity of the international congresses, their size and the possibility of abandoning them:

“I think that this would be a grave error. I speak to scientists from other fields and they express their envy of the fact that we mathematicians have

meetings where the best mathematicians tell us all what the main problems, directions and paradigms in their subjects are; where we honour the winners with the best prizes, and we listen to and discuss their work; where we have group discussions in the hallways on important subjects that our science and our community faces.”

THE BORROMEAN RINGS

At the 2006 congress in Madrid the International Mathematical Union presented its new logo, created by John Sullivan, from the Technical University of Berlin. The logo is based on the Borromean rings which throughout history have symbolised the interconnection and strength through unity. In this case they represent the interlacing and unity of the areas of mathematics and the international mathematical community. An explanatory video by the logos creator can be seen here: <http://www.isama.org/jms/Videos/imu/>.



International Mathematical Union logo.

Zurich offered to host the international congress in 1994 as a compromising, last-minute solution to save the continuity of the series of congresses. Kyoto and Munich were competing for the organisation of the 1990 congress. Opening up to the east was favoured, expecting Munich to organise the 1994 congress, but when the time came this was not to be and, once more, for the third time, Zurich organised the congress.

Four Fields Medals were awarded at the congress to the Belgian mathematician Jean Bourgain from the IHES, Illinois University and the Princeton Institute of Advanced Study, whose work “touches on many central topics of mathematical analysis, where he has opened up new options in problems in which progress came

to a halt a long time ago”; to the French mathematician Pierre-Louis Lions from the Paris-Dauphine University for his “unusual contributions, which cover a variety of areas, from probability theory to partial differential equations, always motivated by applications”; to the French mathematician Jean-Christophe Yoccoz from the University of Paris-Sud in Orsay, who is “*un pur produit et du meilleur cru, du système Français*”, for his contributions to the theory of dynamic systems; and the Russian mathematician Efim Zelmanov from the Universities of Wisconsin and Chicago for “his solution to the restricted Burnside problem”.



The 1994 Fields winners: from top to bottom and left to right, Jean Bourgain, Pierre-Louis Lions, Jean-Christophe Yoccoz and Efim Zelmanov.

The Nevanlinna Prize was awarded to the Israeli computer scientist Avi Wigderson from the Hebrew University of Jerusalem for his contributions to the

mathematical fundamentals of computation and his “impressive results on interactive proofs of zero knowledge”.

Note that the president of the committee that selected the Nevanlinna prize winner was the reputed French mathematician Jacques-Louis Lions, president of the International Mathematical Union at the time and father of the new Fields medallist Pierre-Louis Lions. This gave rise to allegations of nepotism, which the president of the Fields committee, and Fields medallist in his own right David Mumford, still fervently denies today.

Three of the 16 plenary conferences, which were given by two mathematicians, are worth highlighting: ‘Interactions between Ergodic Theory, Lie Groups and Number Theory’, by Marina Ratner, a Russian mathematician from California University in Berkeley; ‘Wavelets and Other Phase Localisation Methods’, by Ingrid Daubechies, a Belgian mathematician from Princeton University, and the final lecture of the congress, ‘Modular Forms, Elliptic Curves and Fermat’s Last Theorem’, where Andrew Wiles presented his proof of Fermat’s last theorem, explaining at the end, as we saw when we covered the age restriction of the Fields Medal, that “one step of the argument is not complete”.



Marina Ratner (left) and Ingrid Daubechies.

The collapse of the Soviet Union left its stamp on the 1994 congress with the presence of participants from many new counties: Armenia, Azerbaijan, Belarus, Bosnia, Croatia, Czech Republic, Slovakia, Slovenia, Estonia, Georgia, Kazakhstan, Latvia, Lithuania, Macedonia, Moldova, Tajikistan, Turkmenistan, Ukraine, Uzbekistan

and, of course, Russia. The participation of many mathematicians from those countries was possible thanks to the financial support of the Soros Foundation and INTAS, the International Association for the Promotion of Cooperation with Scientists from the New Independent States of the former Soviet Union.

Membership by Country

Albania	3	Ireland	7
Argentina	7	Israel	54
Armenia	8	Italy	48
Australia	7	Ivory Coast	5
Austria	9	Japan	236
Azerbaijan	2	Jordan	2
Bahrain	1	Kazakhstan	12
Belarus	6	Korea	8
Belgium	12	Kuwait	1
Bolivia	1	Latvia	8
Bosnia	3	Lesotho	1
Botswana	1	Lithuania	6
Brazil	20	Luxembourg	1
Bulgaria	3	Macedonia	2
Canada	77	Malaysia	5
Chile	3	Maroc	3
China P.R.	33	Mauritania	1
China Thiwan	13	Mexico	16
Colombia	1	Moldova	5
Croatia	6	Mongolia	1
Cuba	2	Netherlands	17
Czech Rep.	17	New Zealand	3
Denmark	15	Nigeria	5
Ecuador	1	Norway	14
Egypt	3	Philippines	8
Estonia	10	Poland	38
Finland	16	Portugal	5
France	165	Romania	28
Georgia	7	Russia	139
Germany	199	Saudi Arabia	2
Greece	11	Senegal	3
Guatemala	4	Singapore	4
Hong Kong	8	Slovakia	6
Hungary	32	Slovenia	3
Iceland	1	South Africa	14
India	29	Spain	29
Indonesia	1	Sri Lanka	1
Iran	23	Sweden	30

List of the participants by country at the Zurich congress in 1994.

MATHEMATICS AND COMPUTERS

The Swiss mathematician Beno Eckmann, honorary president of the congress in 1994, in his welcome speech refined the cocky phrase which was in fashion at the time. "Whether the mathematicians like it or not, the computer is here to stay," into a much more profound sentence: "Whether the computer likes it or not, mathematics is here to stay."

The East

Measured in terms of assistance, the organising of the international congresses in the East – in Kyoto, Beijing and Hyderabad – was a success; but the success was even greater when seen from the point of view of the impact on the local mathematical communities.

In Kyoto in 1990, 4,102 mathematicians attended, which Ludwig Faddeev, a mathematician from Saint Petersburg and president of the International Mathematical Union, considered to be the highest attendance in the history of the congress, thus overlooking the official figures from the Moscow congress in 1966. Of the 4,102 participants, 2,409 were Japanese. The Japanese mathematical community's devotion to the success of the congress went so far that 1,138 of the (Japanese) participants made cash donations, the amount of which exceeded the rest of the subsidies at the congress.

In Beijing in 2002, the attendance record was broken once more (again, excluding Moscow in 1966), with 4,270 participants, of which 1,973 were Chinese. The congress received great support from the Chinese government, reflecting the traditional respect of Chinese society for scientific achievements was conjugated with an interest in mathematics as a means of modernising the country. The presence of the president of the People's Republic of China, Jiang Zemin, at the opening ceremony confirmed its importance.

Attendance at the 2010 congress in Hyderabad was somewhat lower, with 3,000 attendees, half of whom were from India. Organising the congress in a country which, despite its potential for development, still has multitudes living in tremendous poverty and greatly lacking in infrastructure, was a challenge. In fact, the original plan was to hold the congress in Delhi, but in the end it was held in the city of Hyderabad, in the centre of India, which is home to many renowned research centres (the Centre of Cellular and Molecular Biology and the Centre for DNA Fingerprinting, among others). The congress was held in the area of the city that is known as Cyderabad because of the concentration of technology companies. As in China, there was great support from the Indian government (the country's president was at the opening ceremony), which sought to use science, in this case mathematics, to spur on social progress, which is so much a part of Hindu history and tradition.

This eastward movement was more than just fleeting. If we look at the distribution of participants by continent, at the Zurich congress in 1932, Europe and North America made up 83%, while 4% came from Asia. These percentages were similar

at the Berlin congress in 1998. By contrast, in Beijing 56% came from Asia, while those from Europe and North America represented 40%. In Hyderabad, 60% came from Asia. There is more than just physical proximity in these figures.

The Kyoto congress was a perfect combination of tradition and modernity, as the city itself already is. On the one hand, it is the former capital of the country and the spiritual and traditional centre of Japanese culture, while, on the other, it is home to a multitude of renowned innovative companies and scientific centres. The headquarters of the congress, the impressive Kyoto International Conference Hall, provided lecture halls designed as hyper-modern sanctuaries for science. This was the scene for the opening ceremony that was carried out with all the tradition and style of Japanese courtesy, filled with music and courtesan dances.



The Kyoto International Conference Hall.

The president of the congress was the renowned Japanese probability specialist Kiyosi Itô (who received the Gauss Prize in Madrid in 2006). Four Fields Medals were awarded to the Russian mathematician Vladimir G. Drinfeld from the Steklov Institute in Moscow for the “broadness, conceptual richness, technical strength and beauty of his work on quantum groups and one-dimensional Galois groups”; to the New Zealand mathematician Vaughan F. R. Jones from California University in Berkeley, for discovering “surprising relationships between von Neumann algebras and geometric topology”; to the Japanese mathematician Shigefumi Mori from the University of Kyoto for his work on “problems in connection with the classification

problems of algebraic varieties of dimension three”; and to US physicist Edward Witten from the Princeton Institute of Advanced Study for his influence on “the notable rebirth of interaction between mathematics and physics”.

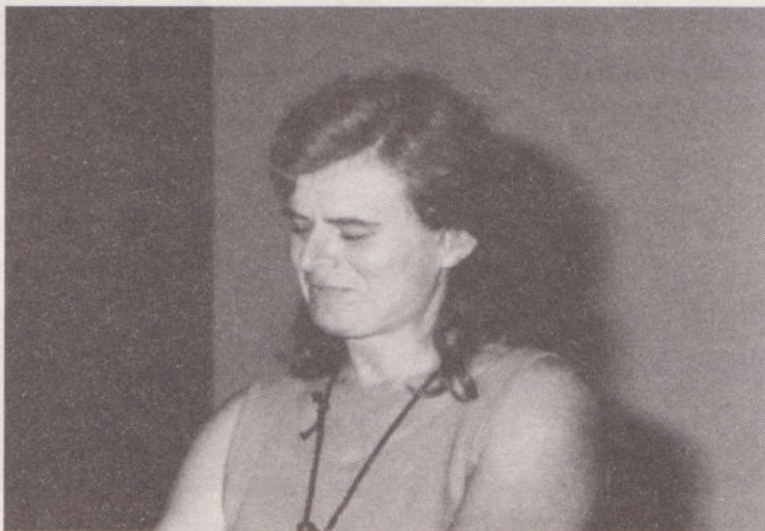
The significant role of mathematical physics can be seen in these prizes. Witten was a physicist, and both Drinfeld and Jones had a very strong scientific relationship with physics.



Vladimir G. Drinfeld, Vaughan F.R. Jones, Shigefumi Mori, Edward Witten.

The Nevanlinna Prize went to the Russian computer scientist and mathematician Alexander A. Razborov, from the Steklov Institute in Moscow, for his “innovative work the lower levels in the complexity of circuits”.

One of the 15 plenary lectures was notable as it was the second given by a woman, after the one at the Zurich congress in 1932 by Emmy Noether: “Applications of Non-Linear Analysis in Topology”, by the US mathematician Karen Uhlenbeck, from the University of Texas in Austin.



Karen Uhlenbeck.

SCIENCE COUNCIL OF JAPAN

The Scientific Council of Japan was created in 1949, after World War II. It is a government organisation formed by renowned scientists, whose objective is to promote scientific development in order to improve the administration, industry and the standard of living. Its existence is one of the factors that explains the economic development of Japan. It is also behind the surprising account of Hikosaburo Komatsu, the president of the congress: He said that when executives of private companies were asked to sponsor the congress they revealed "an appreciation of mathematics because of its important role in the development of the Japanese economy".

Everything at the Beijing congress was done on a large scale. For a start, 5,000 people gathered for the opening ceremony, including participants, authorities and invitees, but they barely filled the enormous main auditorium of The People's Palace, in Tiananmen Square (where the Chinese Communist Party holds its disproportionate meetings). First the president of the congress was chosen as the great Chinese geometrician Shiing-Shen Chern, who was 90 years old at the time and had attended the international congress in Oslo in 1936, 66 years earlier. As was to be expected in 'communist China', between the traditional music, and the speeches there was time to cite Karl Marx and his writings on mathematics and science.



Night view of the main façade of the People's Palace in Beijing.

Once again, the Fields Medals were doubly newsworthy, because of the winners, as always, but also because of their number. As had happened in Vancouver in 1974, only two medals out of the four available were awarded. They went to the French mathematician Laurent Lafforgue, from the Institute of High Scientific Study for an “enormous advance in the Langlands Program, thus showing new connections between the theory of numbers and analysis”; and to the Russian mathematician Vladimir Voevodsky from the Princeton Institute of Advanced Study for “developing new cohomology theories for algebraic varieties, thus providing new visions on the theory of numbers and geometric algebra”. The medals were presented by the president of China himself, Jiang Zemin.



Laurent Lafforgue (left) and Vladimir Voevodsky.

The Nevanlinna Prize was awarded to the Indian mathematician Madhu Sudan of the Massachusetts Institute of Technology for his contributions to “the probabilistically verifiable proofs, to the non-approximability of certain optimisation problems and to error correction codes”.

The inauguration was followed by a large banquet for the 5,000 attendees in the Banquet Hall of the People’s Palace.

The large numbers continued; the congress included 20 plenary conferences, 174 invited lectures in the sections and 1,200 short talks and posters. All in line with the 4,270 participants.

Of all the plenary conferences, there was only one whose title was not in English, that of Fields medallist Lafforgue. Before starting, Lafforgue expressed his regrets about the “Domination of the whole world by one country (whatever its merits),

by one sole culture and by one sole language, which could be very destructive for the diversity of thinking.” Illustrating his point, Lafforgue gave the lecture in English while the slides were written in Chinese and French.

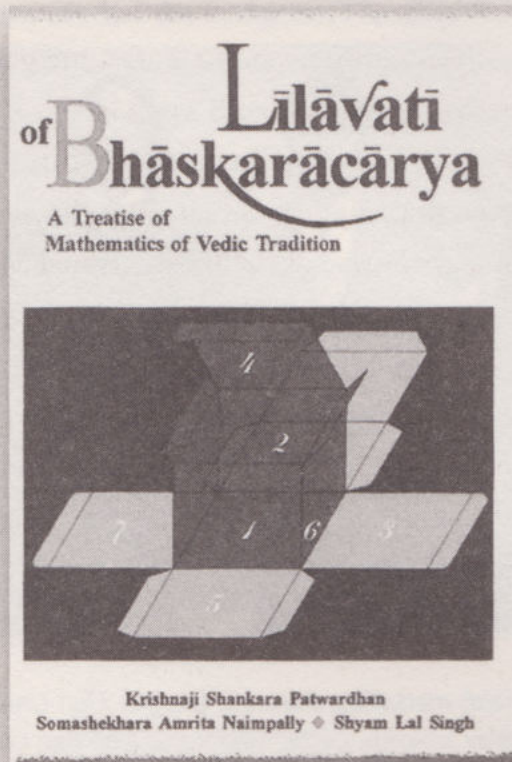
The normalisation of the role of women in international mathematical cooperation took another step, albeit a small one: the first female member was selected for the Administrative Board of the International Mathematical Union. She was the Norwegian mathematician Ragni Piene (whose father, also a mathematician, appears in the photo of the group of participants from the Oslo congress in 1936).

Ludwig Faddeev, a mathematician from Saint Petersburg who had presided over the International Mathematical Union and many of its committees, expressed the feelings of a veteran (he had been attending international congresses since 1962) when he said:

“The main idea of the international congresses is to confirm the unity and universality of mathematics.”

At the Hyderabad congress in 2010 there had to be, and there was, time for remembering India’s rich past in mathematics. It was the president of India, Shrimati Pratibha Devisingh Patil, who paid tribute to the great Hindu mathematicians: Baudhayana, who in the 8th century wrote the *Baudhayana Sulva Sutra*, Vedic texts with precise instructions on geometry and proportion and rules for the construction of altars; Aryabhata, who wrote about subjects in mathematics and astronomy in the 5th century on the work written in verse which bears his name, *Aryabhatiya*; Brahmagupta, who made a name for himself a century later and played an important role in the development of algebra; Bhaskaracharya, a prolific mathematician with a wide field of interest who wrote texts in the 12th century which were used to teach arithmetic and algebra in India for centuries (author of the famous *Lilavati* book, on teaching mathematics). The list would not have been complete without a mention of Srinivasa Ramanujan, the great Indian mathematician and national hero of 20th-century science.

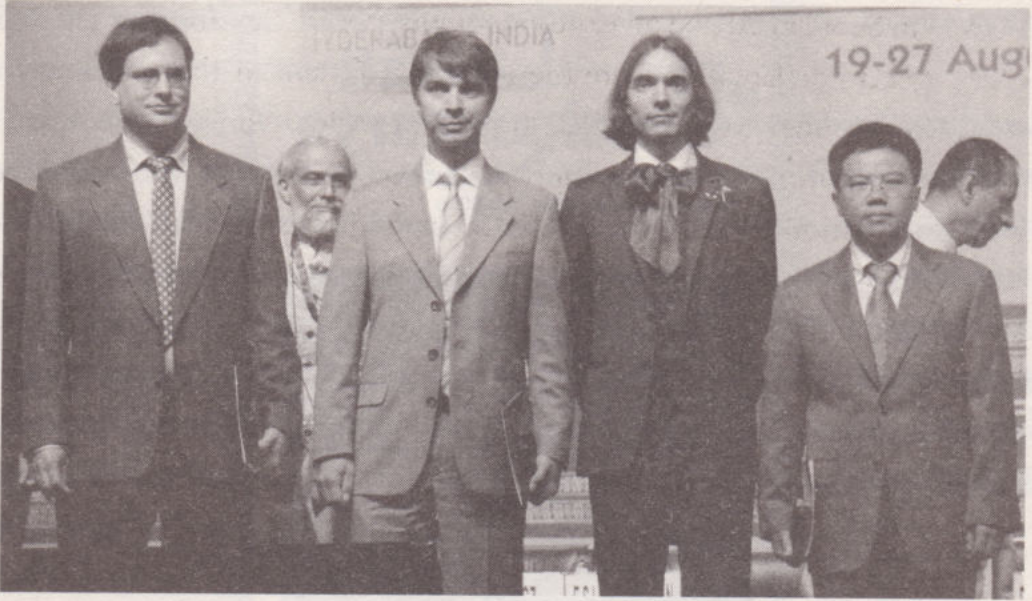
Before starting the presentation of prizes there was time for some traditional verses in Sanskrit, read by the president of India: “Like the crest of the peacock and the jewel of the serpent, mathematics is the rudder of the sciences”, of which you can make what you will. There were also ancient sayings in Telugu, the language of the state of Andhra Pradesh, whose capital is Hyderabad: “He who is good at calculations is a great man.”



Cover of the book Lilavati by Bhaskaracharya.

It was the president of India himself who presented the four Fields Medals at the congress. They went to the Israeli mathematician Elon Lindenstrauss from the Hebrew University of Jerusalem and Princeton University for “his results on ergodic theory and its applications to the theory of numbers”; to the Vietnamese mathematician Ngô Bao Châu from the University of Paris-Sud in Orsay for his “proof of the fundamental lemma in the theory of automorphic forms through the introduction of new algebro-geometric methods”; to the Russian mathematician Stanislav Smirnov from the University of Geneva for “his proof of conformal invariance of percolation and for the two-dimensional Ising model in statistical physics”; and to the French mathematician Cédric Villani, from the Institut Henri Poincaré in Paris, “for his demonstrations of the non-linear Landau damping and convergence to equilibrium of the Boltzmann equation”.

As for the other prizes, the Nevanlinna Prize went to US mathematician Daniel Spielman from Yale University for his work on graphs and algorithms, and the Gauss prize, to the 72-year-old French mathematician Yves Meyer, whose contributions to the theory of wavelets have had a profound impact in many fields of technology, particularly in signal processing.



The 2010 Fields medallists: from left to right, Elon Lindenstrauss, Stanislav Smirnov, Cédric Villani and Ngô Bao Châu.

This congress was one for prizes, as a new prize was awarded and presented for the first time. The Chern Medal, in honour of the Chinese mathematician Shiing-Shen Chern, which is given for a life dedicated to great achievements in mathematics. The peculiarity of this prize is the fact that it is received in two parts: \$250,000 for the winner and \$250,000 for 'mathematical causes' of the winner's choice. It was awarded to the 85-year-old Canadian mathematician Louis Nierenberg, the main founder of the subject of non-linear elliptic curves.

A BOOK ABOUT THE CONGRESSES

The Madrid congress in 2006 was number 25 in the series of the international congresses of mathematicians. This is why the International Mathematical Union chose to celebrate it by organising an exhibition, consisting of a visual chronicle of the congresses and their meaning. It included 500 images from universities and public and private archives. The content of the exhibition was made into the book *Mathematicians of the World, Unite! The International Congress of Mathematicians – A Human Endeavour*. At the Hyderabad congress in 2010 this book was given as the official gift from the International Mathematical Union to the Fields medallists and other prize winners. The author of this book commissioned the exhibition and wrote the book.

Two pieces of news concluded the congress. The first was that the next congress would be held in Seoul in 2014. The Koreans competed with the Brazilian bid to hold the congress in Rio de Janeiro, where there is an important mathematical research centre. The Korean offer to provide 1,000 grants to allow young mathematicians from developing countries to attend tipped the balance in its favour. The second was that the International Mathematical Union chose its president for the period of 2010–2014, who in this case was a woman, the 56-year-old Belgian mathematician Ingrid Daubechies from Princeton University, who gave one of the plenary lectures at the Zurich congress in 1994.

Chapter 8

The Riemann Hypothesis

“Essential progress in the theory of the distribution of prime numbers has lately been made by Hadamard, de la Vallée-Poussin, Von Mangoldt and others. For the complete solution, however, of the problems set us by Riemann’s paper ‘On the Number of Primes Less Than a Given Magnitude’, it still remains to prove the correctness of an exceedingly important statement of Riemann, viz., that the zero points of the function $\zeta(s)$ defined by the series

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \dots$$

all have the real part $\frac{1}{2}$, except the well-known negative integral real zeros. As soon as this proof has been successfully established, the next problem would consist in testing more exactly Riemann’s infinite series for the number of primes below a given number and, especially, to decide whether the difference between the number of primes below a number x and the integral logarithm of x does in fact become infinite of an order not greater than $\frac{1}{2}$ in x .”

This is how David Hilbert explained the problem that we now know as the ‘Riemann hypothesis’ when he presented his list of 23 problems at the international congress of mathematicians in Paris in 1900. It is problem VIII on the list and one of the two that are still unresolved (we have already mentioned that this statement requires clarification).

Likewise, the Riemann hypothesis forms part of the seven ‘Millennium Problems’. According to Enrico Bombieri, who was a Fields Medallist at the 1974 congress held in Vancouver, “In the opinion of many mathematicians the Riemann hypothesis is probably the most important open problem in current mathematics.”

THE MILLENNIUM PROBLEMS

In 1988, the wealthy financier and lover of mathematics Landon T. Clay founded the Clay Mathematics Institute with the purpose of "increasing and disseminating mathematical knowledge". In the year 2000, and as a celebration of the start of the new millennium, the creation of a prize for the resolution of seven mathematical problems of great difficulty and importance was announced. Each solution would be rewarded with \$1 million. A group of reputed mathematicians created the list of problems. One of them – the Poincaré conjecture – has already been resolved by Grigori Perelman. Just as he did with the Fields medal, Perelman rejected his prize (see <http://www.claymath.org/millennium/>).

In what follows, we will try to provide an explanation of the Riemann hypothesis which is as close as possible to the truth, while being suitable for a general public which is unversed in mathematics. This is no easy task, therefore we, both readers and the author, should be patient with it. There are two prerequisites for demonstrating the problem – understanding the idea of an infinite sum and clearly imagining complex numbers (also known as imaginary numbers).

Infinite sums and complex numbers

Let's start by giving sense to an infinite sum with an example (related to the paradox of Achilles and the Tortoise):

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 1.$$

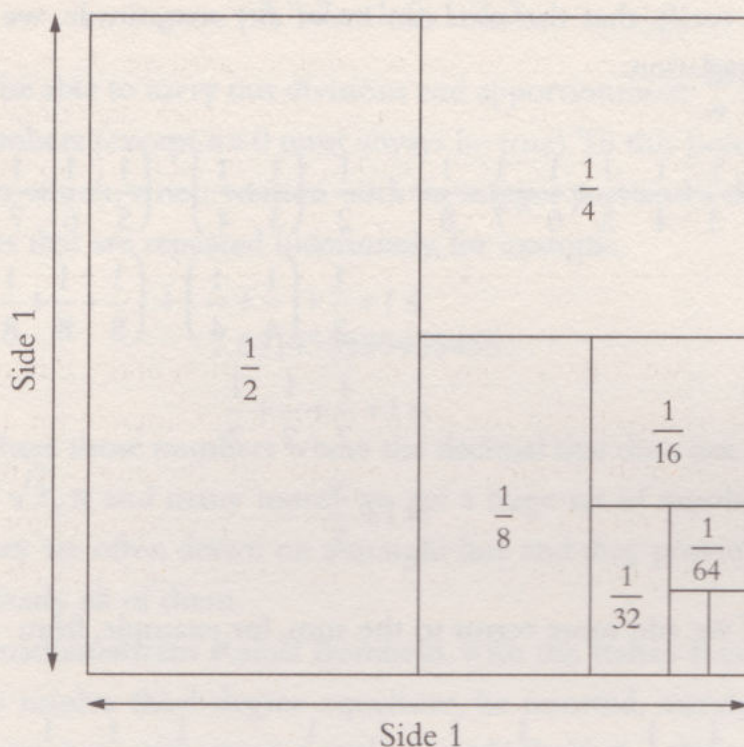
Here the letter n on the 2 means that we take increasingly greater powers of 2. "The infinite sum of 1" means something very specific, namely: that if we add enough terms the result will be a number very close to one.

Let's take a look. If we add the first 33 terms we get (by means of a simple formula, which we are not going to prove now):

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{33}} = \frac{\frac{1}{2} - \frac{1}{2^{33+1}}}{1 - \frac{1}{2}} = 1 - \frac{1}{2^{33}}.$$

Now, if instead of 33 we take the number n of summands, the result will be as close as possible 1, simply by making n very large, as the difference with 1 will be

$\frac{1}{2^n}$, which, evidently gets very small as n increases.



Geometric representation of the infinite sum that gives 1.

There are also infinite sums 'that do not add up to any number', such as:

$$1 - 1 + 1 - 1 + 1 - 1 + \dots - 1 + 1 \dots$$

Let's take a look. Notice that every time we add an even number of terms the result will be 0, as there are as many 1s as there are -1s, while if we add an odd number, the positive and negative terms are grouped in pairs which add up to zero, leaving the last one that will be a 1. Thus, as we progress through the sum the result alternates between 0 and 1, which does not tend towards any number. In this case we say that the infinite sum cannot be realised or that 'it does not converge'.

There is a different and significant case – when the sum becomes infinite, that is, when the sum does not tend towards any number, but becomes increasingly large, exceeding any possible amount. The obvious example is

$$1+2+3+4+\dots+n+\dots$$

But the same can occur even if we add a smaller quantity each time. Let's look at an example (which is also called the *harmonic series*):

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots \quad (1)$$

In order to verify that the sum can be of any magnitude, we carry out the following manipulation:

$$\begin{aligned} 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4} \right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \right) \\ &\geq 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4} \right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \right) \\ &= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\ &= 1 + \frac{3}{2}. \end{aligned}$$

Note that if we add more terms to the sum, for example, from $\frac{1}{9}$ to $\frac{1}{16}$, using the above idea,

$$\frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{15} + \frac{1}{16} \geq \frac{1}{16} + \frac{1}{16} + \cdots + \frac{1}{16} + \frac{1}{16} = \frac{1}{2},$$

we get $1 + \frac{4}{2}$ instead of $1 + \frac{3}{2}$. Thus we see that the result can be as large as we like by simply adding enough terms to the sum. In this case we say that the sum 'diverges to infinity' and we write

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots = \infty.$$

We are going to write the above situations, merely for convenience of expression, thus:

$$\begin{aligned} \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots &= \sum_{n=1}^{\infty} \frac{1}{2^n} = 1 \\ 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots &= \sum_{n=1}^{\infty} \frac{1}{n} = \infty. \end{aligned}$$

Now let's move on to complex numbers. Let's start, like our forefathers, with the counting numbers 1, 2, 3..., which we call the natural numbers. Then we add, in order to be able to carry out commercial debits, the negative numbers (the zero has already been added, thanks to the Indians) and we have:

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

In order to be able to carry out divisions and apportionment $\frac{m}{n}$, where m and n are any two numbers (except $n \neq 0$ must always be true). To this point we have seen all the numbers which, when written with an integer part and a decimal part, the latter has figures that are repeated indefinitely, for example,

$$7.631455455455455\dots$$

Adding to these those numbers where the decimal that does not follow any rule (they do exist: $\sqrt{2}$, π and many more) we get a huge set of numbers that we call *real numbers*. They are often drawn on a straight line and they provide the answer to all problems. Nearly all of them.

When the mathematician Rafael Bombelli, with the Italian Renaissance in full flow, wanted to resolve third-degree equations, he resorted, merely as a technical device, to the square root of negative numbers (which, in principle, cannot exist, as the square of any number is always positive).

It took several centuries, but in the end sense was given to these strange items as the square roots of negative numbers. Hence, *complex numbers* had to be invented.

The least traumatic (but also least evocative) way of understanding them is to see them as pairs of real numbers (a, b) . We can carry out operations on them, adding, subtracting and multiplying by real numbers:

$$(2, 5) + (7, -1) = (9, 4), (3, 5) - (4, 1) = (-1, 4), 3 \cdot (7, -2) = (21, -6).$$

They can also be multiplied together but, in this case, in order for it to work correctly, the method is somewhat convoluted:

$$(2, 5) \cdot (3, 4) = (2 \cdot 3 - 5 \cdot 4, 2 \cdot 4 + 5 \cdot 3) = (-14, 23).$$

Handling complex numbers is simplified if we write them in the following way:

$$s = (a, b) = a \cdot (1, 0) + b \cdot (0, 1) = a + b \cdot i,$$

where $i = (0,1)$ and $(1,0) = 1$. The complex number i is called the *imaginary unit* and it has a surprising property:

$$i^2 = i \cdot i = (0,1) \cdot (0,1) = (0 \cdot 0 - 1 \cdot 1, 0 \cdot 1 + 1 \cdot 0) = (-1,0) = -1.$$

This means that $i^2 = -1$; or put another way, $\sqrt{-1} = i$. Then there is a square root of -1 but it is a complex number.

That leaves two things: use and intuition. The use of complex numbers is absolute, as, thanks to an important result (the fundamental theorem of algebra), all polynomial equations with real coefficients, for example,

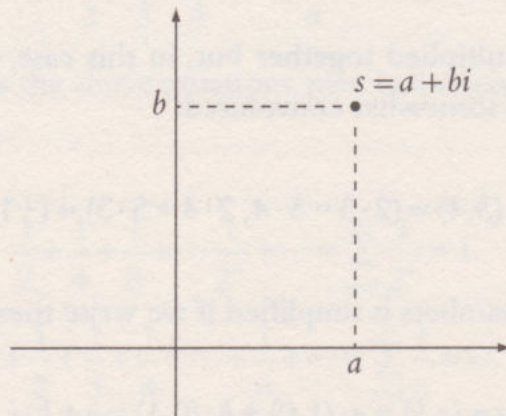
$$9x^6 - 75x^5 - 3x^2 + 8 = 0,$$

have as many *complex* solutions as degrees, in other words, in the example there are 6. The same cannot be said of real roots; for example, $x^2 + 1 = 0$ does not have a real solution but it has two complex solutions.

In order to get a feel for complex numbers, the best thing to do is draw them on a plane with two axes, where the complex number $s = a + bi$ corresponds to the horizontal coordinate point a and vertical coordinate b . The horizontal axis is called the real axis, and the vertical one, the imaginary axis. Hence, if $s = a + bi$, we say that a is the real part of s , and we write $\Re(s) = a$, and that b is the imaginary part of s , and we write $\Im(s) = b$ (both these symbols are strange, but tradition dictates here).

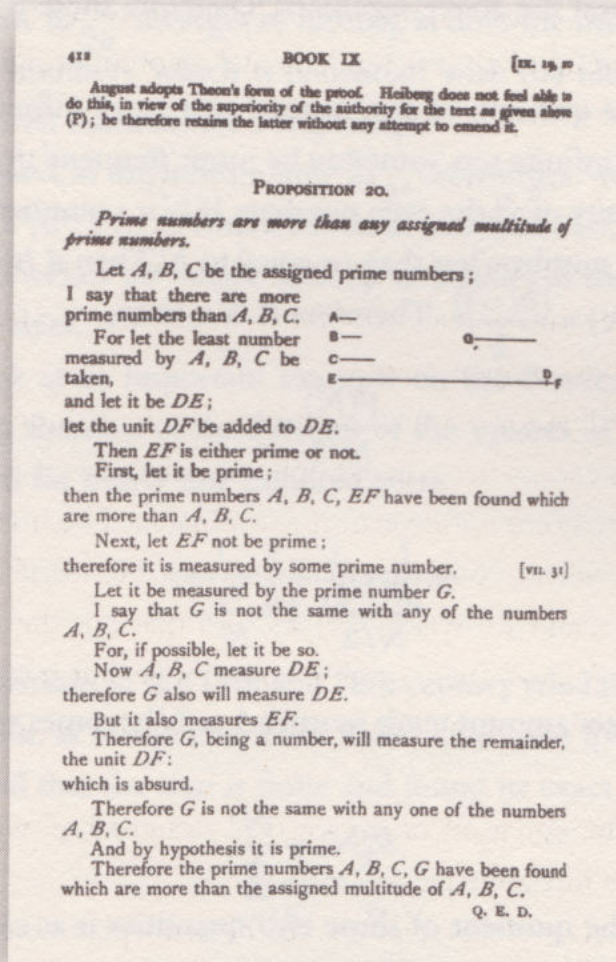
Two examples in view of the above (they are worth considering):

- $\Re(s) = \frac{1}{2}$ is the equation of the vertical straight line which passes through point $\frac{1}{2}$ of the horizontal axis;
- The equation $\Re(s) > 1$ corresponds to the semi-plane located to the right of the vertical straight line which passes through point 1 of the horizontal axis (not including the straight line itself, as on it $\Re(s) = 1$).



Euclid and Euler, the infiniteness of the prime numbers

We have already mentioned that the first proof that there are infinite prime numbers is owed to Euclid, which does not mean that he came up with it himself, but that it appears in his book *Elements*, specifically in Proposition 20 of Book IX.



The Elements by Euclid, proof of the infiniteness of prime numbers. The page shown is from The Thirteen Books of Euclid's Elements, New York, Dover Publications, 1956.

Euclid's argument is beautiful in its simplicity:

If there were only a finite quantity of prime numbers, let's say $p_1, p_2, \dots, p_k$, we form the number

$$N = p_1 \cdot p_2 \cdots p_k + 1.$$

This number N cannot be a prime number, as it is larger than all of $p_1, p_2, \dots, p_k$. But then it must have a prime divisor q . Suppose that q coincides with any of the numbers p_1 , or $p_2 \dots$ or p_k . Then q divides N and also divides $p_1 \cdot p_2 \dots p_k$. As $1 = N - p_1 \cdot p_2 \dots p_k$ is not true, we get that q divides into 1, which is impossible.

Therefore, q is a prime that is not on the initial list, so the list was not complete. A little bit of thought leads to the conclusion that no finite list will be complete, as we can always repeat the above argument. Therefore there is an infinite number of primes.

Let's consider the question of the 'density' of a set of infinite natural numbers. Looking at different infinite sets, some can be more 'frequent' than others. Consider, for example, the density of all the even numbers. If N is a number, we write $P(N)$ for the quantity of even numbers less than or equal to N . Then if N is even, $P(N) = \frac{N}{2}$, and if N is odd, $P(N) = \frac{(N-1)}{2}$. Therefore if N is even,

$$\frac{P(N)}{N/2} = 1,$$

and if N is odd,

$$\frac{P(N)}{N/2} = 1 - \frac{1}{N}.$$

Note that the latter amount tends towards 1 as N becomes very large. The above can be written as

$$P(N) \sim \frac{N}{2},$$

which means that the quotient of those two quantities is as close as possible to 1; when N is sufficiently large,

$$\frac{P(N)}{N/2} \sim 1.$$

So we can think of the totality of the numbers and the even numbers as 'comparable', that is, the phrase "there are half the number of even numbers as there are numbers" makes sense (despite the fact that they are both infinite sets). If we were to repeat the process for multiples of 3, we would get $\frac{N}{3}$, and for multiples of 5, we would get $\frac{N}{5}$.

In order to compare infinite sets of numbers, in other words, to consider their density, we can use infinite sums. If we write the infinite sum of the inverses of the even numbers, we get:

$$\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = \infty.$$

The infinite sum $\sum \frac{1}{2n}$ diverges to infinity, as does the infinite sum $\sum \frac{1}{n}$ of the inverses of all the numbers, which is consistent with the idea that all the natural numbers and the even numbers are comparable.

On the other hand, as the infinite sum $\sum \frac{1}{2^n}$ converges, we can deduce that the powers of two, $\{2, 2^2, 2^3, 2^4, \dots\}$, which still form an infinite set, are 'few' enough that the series of their inverses no longer adds up to infinity, as the sum of the inverses of all the numbers does.

Let's take a look at an important example on the density of infinite sets. The problem of finding the sum of the inverses of the squares of the numbers was an unresolved problem for nearly one hundred years.

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$$

Many mathematicians in the 17th and 18th century tried to solve it. It is known as the Basel problem, as it was Leonhard Euler, who then lived in that Swiss city, who in 1735 proved that the sum is finite and found its exact value:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \quad (2)$$

Apart from being astounded at the result of the sum, we can draw the same conclusion as we did previously for the powers of two: the squares of the numbers are 'few' enough within the infiniteness of all the numbers.

In his book *Introductio in Analysin Infinitorum*, published in 1748, Euler proved an important formula, which he obtained by manipulating the expression for numbers in prime factors, in a simple, but ingenious way; it is known as the Euler product formula:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - \frac{1}{p^s}} \right), \quad (3)$$

where the symbol, similar to the symbol for the sum, means that a product of infinite factors is carried out. By placing a small p under the product symbol we are indicating that there is a factor of that form for every prime number, including all of them. Let's take a look at an example for $s = 2$,

$$\prod_p \frac{1}{\left(1 - \frac{1}{p^2}\right)} = \frac{1}{\left(1 - \frac{1}{2^2}\right)} \cdot \frac{1}{\left(1 - \frac{1}{3^2}\right)} \cdot \frac{1}{\left(1 - \frac{1}{5^2}\right)} \cdot \frac{1}{\left(1 - \frac{1}{7^2}\right)} \dots$$

With the above formula (3), Euler was able to improve the result of infiniteness of the prime numbers by adding information on their 'density':

$$\sum_p \frac{1}{p} = \infty,$$

that is to say, the infinite sum of the inverses of the prime numbers diverges to infinity. Given the interpretations we have seen above, we can deduce from here that the density of the prime numbers among all the numbers is 'high'. Then the question is, how high is the density of the prime numbers?

Gauss and the distribution of the primes

In 1792, Gauss, at the age of 15, based on empirical observation (calculating and referring to tables of prime numbers) and his fine mathematical intuition, came to the conclusion that the density of the prime numbers must be governed by the logarithm in the following specific way: if we call the number of prime numbers lower than N , $\Pi(N)$, then:

$$\Pi(N) \sim \frac{N}{\log N}. \quad (4)$$

Certain work by the French mathematician Adrien-Marie Legendre in around 1800 was in the same area, but it did not provide proof of the result (Gauss' ideas were common knowledge first because those of Legendre were not published until more than 50 years later).

The first positive result was obtained by the Russian mathematician Pafnuty Lvovich Tchebyshev in 1850 when he proved that

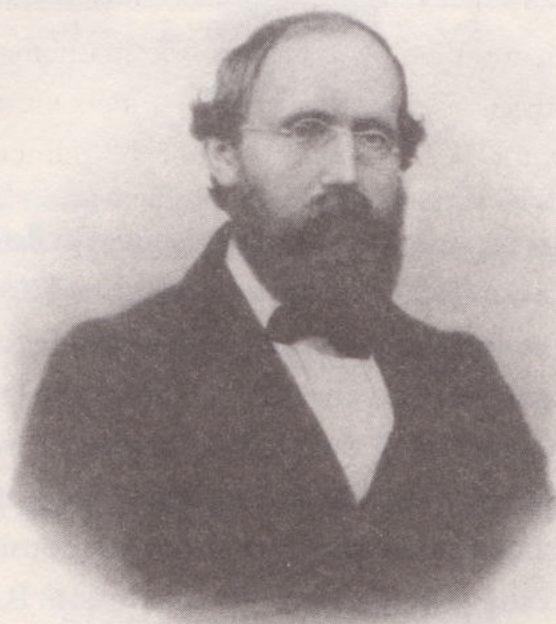
$$0,89 < \frac{\Pi(N)}{N/\log N} < 1,11,$$

when N is very large, which can be interpreted to mean that the “relative error” in the approximation of $\Pi(N)$ with $N/\log N$ is less than 11%.

It was the French mathematician Jacques Hadamard and the Belgian mathematician Charles de la Vallée Poussin who, in 1896, independently proved the result (4), known as the *prime number theorem*, which establishes the asymptotic ‘distribution’ of the prime numbers within the natural numbers. The adjective ‘asymptotic’ alludes to the fact that what it proves, as it involves a limit, establishes a relationship between $\Pi(N)$ and $N/\log N$ for very high values of N .

Riemann and the zeta function

The German mathematician Bernhard Riemann, in the 1859 article which Hilbert quoted, “Über die Anzahl der Primzahlen unter einer gegebenen Grösse”, studied the distribution of prime numbers in great detail. To do so, he exploited a series of theoretical techniques and tools which had not been used in the field previously. The article is short, at just eight pages, but it has left huge mathematical consequences in its wake.



Bernhard Riemann.

Riemann started with the Euler product formula (3), into which he introduced the innovation of complex numbers, thus creating a function, which has come to be known as the *Riemann zeta function*:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}. \quad (5)$$

Note that the function is defined by an infinite sum and that each term is the sum of the inverse of n^s , where n is a natural number, but s is a complex number. For example, 7^{2+3i} , could appear. We are not going to get into the technical details of this operation here, as it is not relevant to the discussion which follows.

As the ζ function is defined by an infinite sum, it will only make sense for those complex numbers s which make for a converging infinite sum. For example, we know that the harmonic series (1) diverges to infinity, which, expressed in terms of the zeta function, means that

$$\zeta(1) = \infty.$$

We also know that the sum of the inverses of the squares of all the natural numbers is convergent and we know its sum (2), because Euler found it, which, expressed in terms of the zeta function means that:

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Riemann verified that the infinite sum which he used to define the zeta function was convergent for all the complex numbers on semi-plane $\Re(s) > 1$ (which we spoke about when we looked at complex numbers). This is simple to prove.

What was more ingenious was the way in which he proved that the zeta function could be calculated for all the complex numbers, even though in order to do so he could no longer use the infinite sum (5). In reality it cannot be calculated for all the complex numbers, as when we consider $s = 1$, we know that $\zeta(1) = \infty$, and this cannot be changed. In conclusion: Riemann proved that the zeta function, $\zeta(s)$, could be calculated for all the complex numbers, except where $s = 1$, where it is infinite.

The zeros of the zeta function

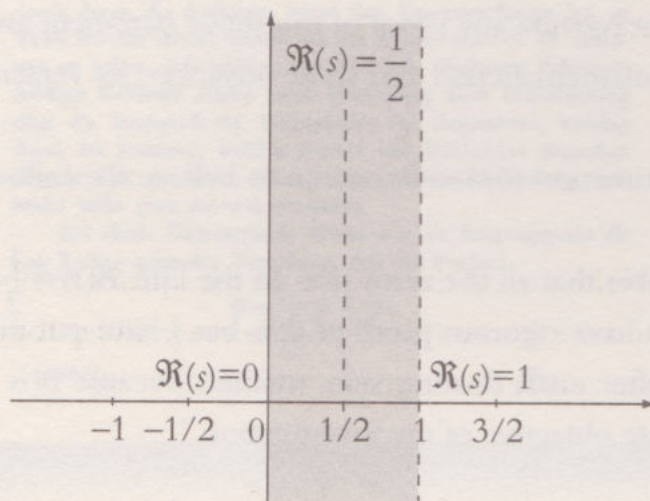
A zero of the zeta function is a complex number s where

$$\zeta(s) = 0.$$

There are some zeros of the zeta function, which are called trivial zeros, in the negative even integers, in other words,

$$\zeta(-2) = 0, \zeta(-4) = 0, \zeta(-6) = 0, \dots$$

But it is the remaining ones, called non-trivial zeros, which are interesting. Riemann proved that non-trivial zeros of the zeta function are in the band of the plane between the vertical straight lines $\Re(s) = 0$ and $\Re(s) = 1$. And this is where the important *critical line* $\Re(s) = \frac{1}{2}$ appeared for the first time.



If we take any real number T , we think of the zeros of the zeta function which are on the line $\Re(s) = \frac{1}{2}$ and whose imaginary part is between 0 and T . That is:

$$\zeta\left(\frac{1}{2} + ti\right) = 0, \text{ being } 0 \leq t \leq T.$$

We will write the quantity of zeros of this type as follows:

$$\#\left\{s = \frac{1}{2} + ti : 0 \leq t \leq T, \zeta(s) = 0\right\}.$$

No formula is known for this quantity, but Riemann wrote an asymptotic estimation of it in his article, specifically:

$$\#\left\{s = \frac{1}{2} + ti : 0 \leq t \leq T, \zeta(s) = 0\right\} \sim \frac{T}{2\pi} \log \frac{T}{2\pi} - \frac{T}{2\pi}. \quad (6)$$

That is, if we calculate the quotient of the previous quantities, it will be as close to 1 as possible, as long as T is sufficiently large

$$\frac{\#\left\{s = \frac{1}{2} + ti : 0 \leq t \leq T, \zeta(s) = 0\right\}}{\frac{T}{2\pi} \log \frac{T}{2\pi} - \frac{T}{2\pi}} \sim 1.$$

Riemann did not provide any proof of the above statement and to date, despite the work of many mathematicians and some advances, its veracity (or falsity) has not been proven.

The previous estimation (6) led Riemann to believe the following:

“It is very probable that all the zeros [are on the line $\Re(s) = \frac{1}{2}$]. One would of course like to have rigorous proof of this, but I have put aside the search for such proof after some fleeting vain attempts because it is not necessary for the immediate objective of my investigation.”

In other words, Riemann believed it to be very possible that all the non-trivial zeroes of the zeta function were on the critical line $\Re(s) = \frac{1}{2}$. This statement has gone down in history and is today known as the ‘Riemann hypothesis’.

To conclude: In principle, the Riemann hypothesis was a lateral statement within the article, the objective of which was different, that of finding a formula for the number of primes smaller than a given quantity, $\Pi(N)$. In his article, Riemann gave a complicated expression for $\Pi(N)$, which involves infinite sums. We now know that the Riemann hypothesis allows, and is in fact equivalent to, a precise estimation of $\Pi(N)$.

Riemann died at the age of 39 in 1866. More than 150 years have passed since he wrote his hypothesis. The problem is still unsolved.

Bericht

über die

zur Bekanntmachung geeigneten Verhandlungen
der Königl. Preufs. Akademie der Wissenschaften
zu Berlin

im Monat November 1859.

Vorsitzender Sekretar: Hr. Encke.

3. Nov. Gesamtsitzung der Akademie.

Hr. Steiner las über einige allgemeine Bestimmungsarten der Curven und Flächen zweiter Ordnung und daraus folgenden Sätzen.

Hierauf trug Hr. Kummer folgende von Hrn. Riemann, Correspondenten der Akademie, mittelst eines an den Sekretar Hrn. Encke gerichteten Schreibens vom 19. October d. J. eingesandte Mittheilung „über die Anzahl der Primzahlen unter einer gegebenen Gröfse“ vor:

Meinen Dank für die Auszeichnung, welche mir die Akademie durch die Aufnahme unter ihre Correspondenten hat zu Theil werden lassen, glaube ich am besten dadurch zu erkennen zu geben, daß ich von der hiedurch erhaltenen Erlaubniß baldigst Gebrauch mache durch Mittheilung einer Untersuchung über die Häufigkeit der Primzahlen; ein Gegenstand, welcher durch das Interesse, welches Gauss und Dirichlet demselben längere Zeit geschenkt haben, einer solchen Mittheilung vielleicht nicht ganz unwerth erscheint.

Bei dieser Untersuchung diene mir als Ausgangspunkt die von Euler gemachte Bemerkung, daß das Product

$$\prod \frac{1}{1 - \frac{1}{p^s}} = \sum \frac{1}{n^s},$$

[1859.]

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Page from Riemann's article.

Bibliography

The original minutes from all the international congresses can be found on the International Mathematical Union's web site:

<http://www.mathunion.org/ICM/>

ALBERS, D. J., ALEXANDERSON, G.L.; REID, C., *International Mathematical Congresses. An Illustrated History, 1893-1986*, Berlin, Springer Verlag, 1986.

<http://www.mathunion.org/ICM/History/history.ocr.pdf>

CURBERA, G.P., *Mathematician of the World, Unite! The International Congress of Mathematicians - A Human Endeavor*. Wellesley, AK Peters, 2009.

EDWARDS, H.M., *Riemann's Zeta Function*, New York, Dover, 1974.

LEHTO, O., *Mathematics Without Borders. A History of the International Mathematical Union*, Berlin, Springer Verlag, 1998.

MONASTYRSKY, M., *Modern Mathematics in the Light of the Fields Medal*, Wellesley, AK Peters, 1998.

NASAR, S., GRUBER, D., *Manifold Destiny: A Legendary Problem And The Battle Over Who Solved It*. *The New Yorker*, 26 August, 2006.

WIGNER, E.P., *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*. *Communications on Pure and Applied Mathematics* (1960), vol. 13, pg. 1-14.

YANDELL, B.H., *The Honors Class: Hilbert's Problems and Their Solvers*, Wellesley, AK Peters, 2002.

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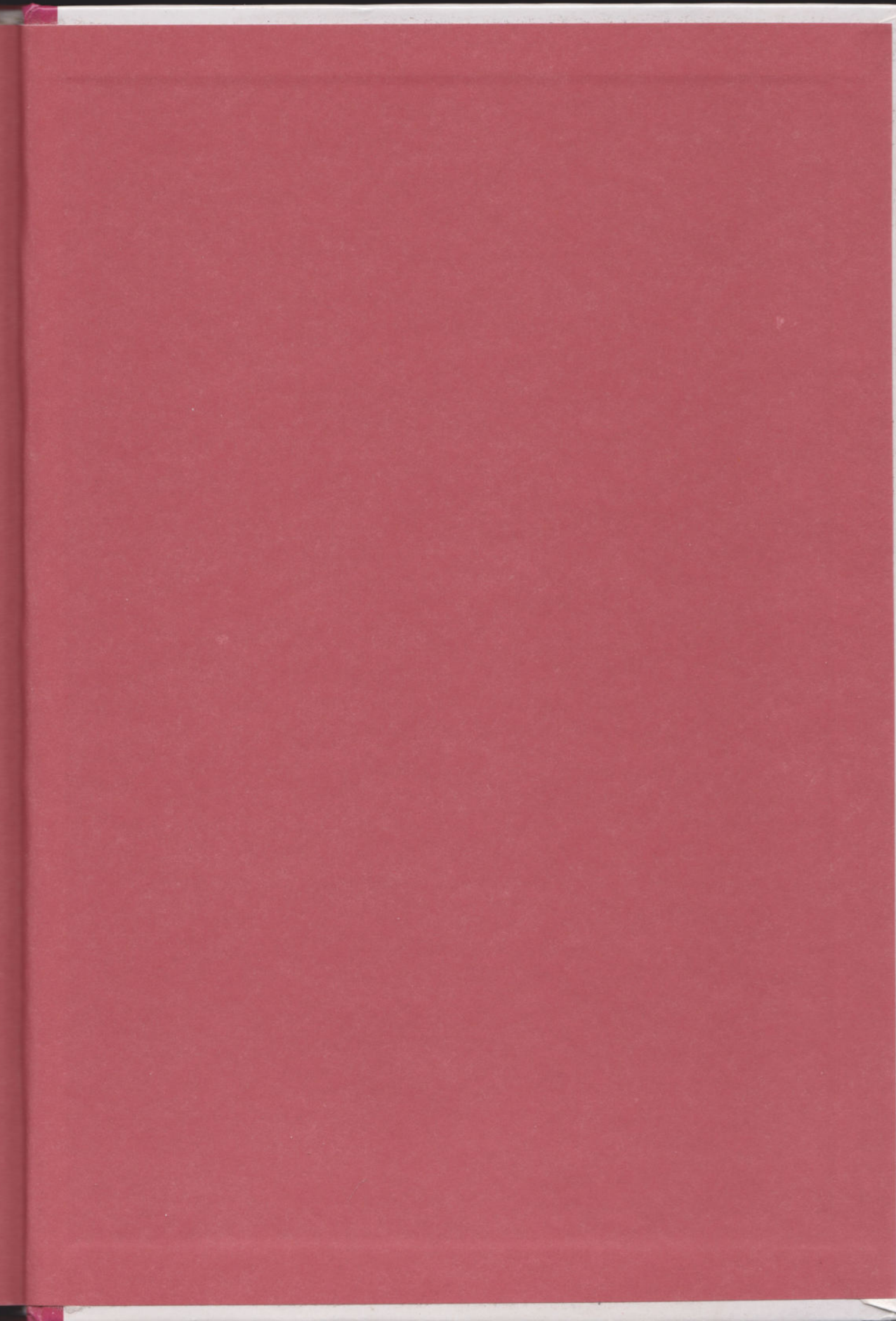
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Mathematical Network

International groups and congresses

Since 1897, every four years the world's mathematical élite gather at international congresses where they present and discuss the most significant advances in the discipline. The history of these congresses reveals to us the extent to which mathematics, often seen as the result of individual contributions from a few geniuses, is, in fact, largely the result of the interaction between like-minded people, who pool their knowledge for the benefit of all humanity.